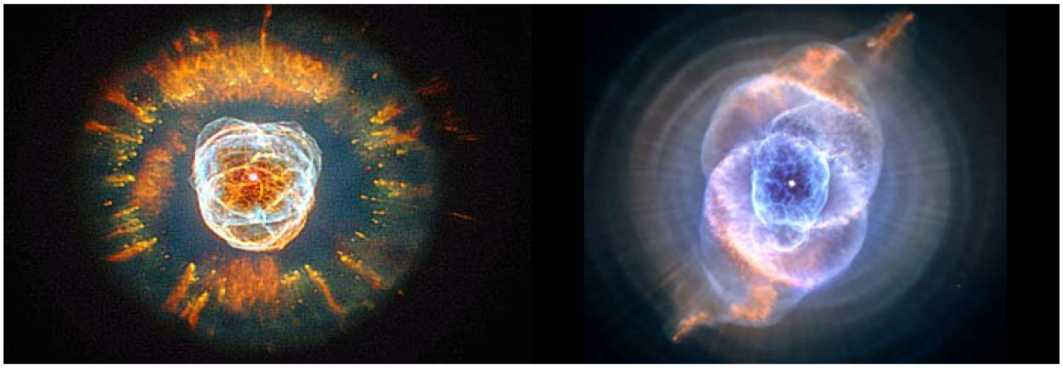
The Electrodynamic Origin of the Force of Gravity—Part 1
($F = G\, m_1\, m_2 / r^2$)

Charles W. Lucas, Jr.
29045 Livingston Drive
Mechanicsville, MD 20659-3271
bill@commonsensescience.org

Abstract. The force of gravity is shown to be a small average residual force due to the fourth order terms in v/c of the derived universal electrodynamic contact force between vibrating neutral electric dipoles consisting of atomic electrons vibrating with respect to protons in the nucleus of atoms. The derived gravitational force has the expected radial term plus a new non-radial term. From the radial term the gravitational mass can be defined in terms of electrodynamic parameters. The non-radial term causes the orbits of the planets about the sun to spiral about a circular orbit giving the appearance of an elliptical orbit tilted with respect to the equatorial plane of the sun and the quantization of the orbits as roughly described by Bode's law. The vibrational mechanism that causes the gravitational force is shown to decay over time giving rise to numerous phenomena, including the expansion of the planets (including the earth) and moons in our solar system, the cosmic background radiation, Hubble's red shifts versus distance (due primarily to gravitational red shifting), Tifft's quantized red shifts (Bode's law on a universal scale), Tifft's measured rapid decay of the magnitude of red shifts over time, the Tulley-Fisher relationship for luminosity of spiral galaxies, the unexpected high velocities of the outer stars of spiral galaxies, and Roscoe's observed quantization of the luminosity and size (Bode's law) of 900 spiral galaxies. Arguments are given that this derived law of gravity is superior to Newton's Universal Law of Gravitation ($F = G m_1 m_2 / r^2$) and Einstein's General Relativity Theory ($G_{\mu\nu} = -8\pi G/c^2\, T_{\mu\nu}$).

Introduction. In the past natural philosophers and astronomers thought that we lived in a disconnected universe consisting of the Milky Way Galaxy with a vacuum or void between the stars and other objects. In recent times astronomers have discovered that space is filled with charged particles forming dynamic plasmas. From Figures 1 and 2 we can see that there are magnetic fields stretching across vast intergalactic distances in these plasmas.



Figures 1 and 2. Plasma Structure of Galaxies Controlled by Magnetic Fields

These magnetic fields appear as filaments or threads extending for huge distances, like millions of light years, connecting things together. Stars appear as something like streetlights along the fibers of the electric universe. This is very different from the universe of Isaac Newton's day which consisted primarily of empty space with astronomical bodies orbiting one another due to the action-at-a-distance gravitational force. The electrodynamic plasma force is 10^{40} times as strong as the gravitational force and appears to play a more dominant role in the universe than the gravitational force as it connects things together.

Einstein's General Relativity Theory, the replacement theory for Newton's theory of gravity, has run into great difficulty in these days. It cannot explain the role of electrodynamic plasmas in the organization of the universe. Furthermore, it cannot explain the observed structure of spiral galaxies without inventing dark matter, such that the universe consists of 90-95% dark matter which cannot be observed in the laboratory like other types of matter. In addition, in order to explain the expansion of the universe using relativity theory, it has been necessary to invent dark energy which can not be observed directly either.

Instead of pursuing these problematic approaches to a disconnected universe, it would seem better to consider an electrodynamic approach for a connected universe in agreement with Mach's Principle. Ever since the discovery of Coulomb's electrostatic force law [1] between charges q_1 and q_2 and Newton's universal gravitational force law [2] between masses m_{g1} and m_{g2}

$$F_{Coulomb} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{\vec{R}^2}$$

$$F_{Gravity} = G\frac{m_{g1}m_{g2}}{\vec{R}^2} \quad \text{where } \vec{R} = \vec{r}_2 - \vec{r}_1 \qquad (1)$$

scientists have imagined that there might be a relationship between electrodynamics and gravity, because the form of the force laws is identical.

The philosopher Poincaré [3] argued from logic that any two fundamental forces that employed the same fundamental constants or the same mathematical form were not both fundamental. The most advanced theory of gravity, Einstein's General Relativity Theory [4] as represented in equation (2), is expressed in terms of c the velocity of light which is the fundamental constant of electrodynamics.

$$G_{\mu\nu} = -\frac{8\pi}{c^2} G\, T_{\mu\nu} \tag{2}$$

Einstein also said in his Principle of Equivalence [5] that gravity and inertia are intimately related. Poincaré suggested that electrodynamics might be the origin of both the gravitational force as well as the related inertial force, because both force laws have a similar form and both involve the velocity of light which is the fundamental constant of electrodynamics.

According to the Lorentz force law combined with Ampere's law of induction for a moving charge $\left(\vec{B}_i = \left(qv/c\right)\vec{E}_o\right)$ is

$$\vec{F} = q\vec{E}_o + \frac{\vec{v}\times\vec{B}_i}{c} = q\vec{E}_o + \frac{\vec{v}\times\left(\frac{q\vec{v}}{c}\times\vec{E}_o\right)}{c} \approx q\vec{E}_o\left(1+\frac{v^2}{c^2}\right) \tag{3}$$

where the induced $\mathbf{B}_i$ field adds a term to the static Coulomb field E_o proportional to v^2/c^2. The free electron drift velocity in conductors is typically on the order of 0.03 meters per second (m/sec) such that

$$\frac{v^2}{c^2} \approx \left(\frac{3\times10^{-2}\,\text{m/sec}}{3\times10^{8}\,\text{m/sec}}\right)^2 \approx 10^{-20} \tag{4}$$

Now the gravitational force is approximately 10^{-40} times smaller than the static Coulomb electric force, suggesting that the gravitational force might be a higher order v^4/c^4 term multiplying the static Coulomb force.

The derived universal electrodynamic contact force, [6, 7] as given in equation (5) below gives terms of this sort. In the past such higher order terms in the electrodynamic force were never fully investigated.

$$\vec{F}\left(\vec{R},\vec{V},\vec{A},\ldots\right)=\frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\left(1-\vec{\beta}^2\right)\vec{R}+\dfrac{2\vec{R}^2}{c^2}\vec{A}}{\vec{R}^2\left[\vec{R}^2-\dfrac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^2}{\vec{R}^2}\right]^{1/2}}$$

$$-\frac{q_1 q_2}{4\pi\varepsilon_o}\left(1-\vec{\beta}^2\right)\frac{\left[\left(\vec{\beta}\cdot\vec{R}\right)\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)+\left(\vec{R}\cdot\vec{R}\right)\vec{R}\times\left(\vec{R}\times\dfrac{\vec{A}}{c^2}\right)\right]}{\vec{R}^2\left[\vec{R}^2-\dfrac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^2}{\vec{R}^2}\right]^{3/2}} \qquad (5)$$

For $\vec{A}=0$

$$\vec{F}\left(\vec{R},\vec{V},\vec{A},\ldots\right)=\frac{q_1 q_2}{4\pi\varepsilon_o\vec{R}^2}\frac{\left(1-\beta^2\right)\hat{R}}{\left(1-\beta^2\sin^2\theta\right)^{1/2}}-\frac{q_1 q_2}{4\pi\varepsilon_o\vec{R}^2}\frac{\left(1-\vec{\beta}^2\right)\left(\vec{\beta}\cdot\hat{r}\right)\hat{r}\times\left(\hat{r}\times\vec{\beta}\right)}{\left(1-\beta^2\sin^2\theta\right)^{3/2}}$$

where $\vec{R}=\vec{r}_2-\vec{r}_1 \quad \vec{V}=\dfrac{d\vec{R}}{dt}=\dfrac{d\vec{r}_2}{dt}-\dfrac{d\vec{r}_1}{dt} \quad \vec{A}=\dfrac{d\vec{V}}{dt}=\dfrac{d^2\vec{R}}{d^2t}=\dfrac{d^2\vec{r}_2}{d^2t}-\dfrac{d^2\vec{r}_1}{d^2t}$

In the universal force the first acceleration **A** term above gives rise to Newton's Second Law (**F**=m**a**) for the force of inertia, and the second acceleration **A** term gives rise to absorption and emission of electromagnetic radiation or light. This universal contact force in the limit of constant velocity, i.e. $\mathbf{A}=0$ corresponds, mathematically speaking, to the action-at-a-distance covariant relativistic electrodynamic force based on Maxwell's equations [8, p. 555; 9, p. 560]. The derivation to follow is valid for both force laws, since they are identical in the limit $\mathbf{A}=0$.

Origin of Gravitational Forces. The force of gravity is normally measured between neutral charge bodies. Since the time of Newton, scientists have learned that the neutral atom consists of electrons, protons, and neutrons. Even elementary charged particles such as the neutron and proton consist of smaller components of positive and negative charge such as quarks or charge fibers. This work explores the possibility that the attractive force of gravity is due to a small residual electrodynamic force between vibrating or oscillating neutral electric dipoles. These dipoles could be inside an elementary particle or an atom, but for this work we will assume that the primary effect is from protons and electrons in atoms.

Assuming constant velocity $\mathbf{A}=0$ and $\boldsymbol{v}/c=\boldsymbol{\beta}$ is very small one may expand the terms in the equation (5) for the universal electrodynamic force for the radial term using the binomial expansion and keeping only terms to order $\boldsymbol{\beta}^4$ to obtain

$$\vec{F}(\vec{R},\vec{V}) \approx \frac{q_1 q_2}{4\pi\varepsilon_o R^2}\left(1-\vec{\beta}^2\right)\left(1+\frac{\beta^2\sin^2\theta}{2}+\frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{2}\beta^4\ \sin^4\theta+\ldots\right) \tag{6}$$

$$\begin{aligned}
&-\frac{q_1 q_2}{4\pi\varepsilon_o R^2}\left(\vec{\beta}\cdot\hat{R}\right)\hat{R}\times\left(\hat{R}\times\vec{\beta}\right)\left(1-\beta^2\right)\left(1+\frac{3}{2}\beta^2\sin^2\theta\right)+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{2}\beta^4\sin^4\theta+\ldots\\
&\approx\frac{q_1 q_2}{4\pi\varepsilon_o R^2}\hat{R}\left[1-\frac{\beta^2}{2}-\frac{\beta^2}{2}\cos^2\theta-\frac{\beta^4}{8}-\frac{\beta^4}{4}\cos^2\theta+\frac{3\beta^4}{8}\cos^4\theta+\ldots\right]\\
&-\frac{q_1 q_2}{4\pi\varepsilon_o R^2}\left(\vec{\beta}\cdot\hat{R}\right)\hat{R}\times\left(\hat{R}\times\vec{\beta}\right)\left[1+\frac{\beta^2}{2}-\frac{3\beta^2}{2}\cos^2\theta-\frac{3\beta^4}{8}-\frac{9\beta^4}{4}\cos^2\theta+\frac{15\beta^4}{8}\cos^4\theta+\ldots\right]
\end{aligned}$$

Consider the force between two neutral electric dipoles each consisting of positive protons and negative electrons. If we label the charges q_{1+}, q_{1-}, q_{2+}, and q_{2-}, the total force between the two neutral dipoles is given by equation (7)

$$\vec{F} = \vec{F}_{2+,1+} + \vec{F}_{2+,1-} + \vec{F}_{2-,1+} + \vec{F}_{2-,1-} \tag{7}$$

where the dipoles are defined in Figure 3 for hydrogen atoms where the larger toroid is the electron and the smaller toroid is the proton.

$$\begin{aligned}
&\vec{r}_{1+2+} = \vec{r}_{2+} - \vec{r}_{1+} \qquad \omega_1 = 2\pi f_1 \qquad \omega_2 = 2\pi f_2\\
&\vec{r}_{1-2+} = \vec{r}_{2+} - \vec{r}_{1-} - \vec{A}_1\cos(\omega_1 t + \phi_1) \qquad A_1 f_1 = v_1\\
&\vec{r}_{1+2-} = \vec{r}_{1+} - \vec{r}_{2-} - \vec{A}_2\cos(\omega_2 t + \phi_2) \qquad A_2 f_2 = v_2\\
&\vec{r}_{2-1-} = \vec{r}_{2-} - \vec{r}_{1-} - \vec{A}_2\cos(\omega_2 t + \phi_2) - A_1\cos(\omega_1 t + \phi_1)
\end{aligned}$$

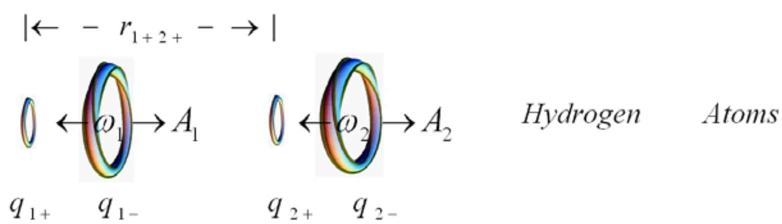


Figure 3. Oscillations of Electrons in Neutral Dipoles

The amplitudes of oscillation A_1 and A_2 should be on the order of the size of the atom $\approx 10^{-10}$ meter or less. From the wave equation $Af = c$, the frequencies of oscillation ω_1 and ω_2 must be in the microwave range $\approx 10^{10}$ per second. At $t = 0$ the amplitude of vibration is maximum as represented by $\cos(\omega t + \varphi)$.

In order to simplify the calculations assume that the positively charged proton is much more massive than the negatively charged electron, such that the vibratory motion of the dipole can be considered as due primarily to the motion of the electron. Since the mass of the proton is 1836 times the mass of an electron, this is a reasonable approximation.

In order to calculate a quantity comparable to the force of gravity, it will be necessary to perform a number of averages. Each oscillating dipole has a different phase that must be averaged over. Each oscillating dipole may have a different physical orientation that needs to be averaged over for all the dipoles in the material body. In order to obtain a time independent value it will be necessary to perform a time average on each of the oscillating dipoles. Thus the force to be compared with gravity is

$$\vec{F}\left(\vec{R},\vec{V}\right)=\frac{1}{T_1}\int_0^{T_1}dt_1\ \frac{1}{T_2}\int_0^{T_2}dt_2\ \frac{1}{2\pi}\int_0^{2\pi}d\varphi_1\ \frac{1}{2\pi}\int_0^{2\pi}d\varphi_2\ \frac{1}{2\pi}\int_0^{2\pi}d\varphi \\ \times\frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\ \frac{1}{2\pi}\int_0^{2\pi}d\varphi_1\ \vec{F}\left(R,\theta,\varphi,\vec{A}_1,\omega_1,\varphi_1,t_1,\vec{A}_2,\omega_2,\varphi_2,t_2,V\right) \tag{8}$$

For simplicity consider that there are two collections of dipoles to average over and that the two collections have spherical symmetry. In this case the integral over φ becomes just 2π giving

$$\vec{F}\left(\vec{R},\vec{V}\right)=\frac{1}{T_1}\int_0^{T_1}dt_1\ \frac{1}{T_2}\int_0^{T_2}dt_2\ \frac{1}{2\pi}\int_0^{2\pi}d\varphi_1\ \frac{1}{2\pi}\int_0^{2\pi}d\varphi_2 \\ \times\frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\ \vec{F}\left(R,\theta,\varphi,\vec{A}_1,\omega_1,\varphi_1,t_1,\vec{A}_2,\omega_2,\varphi_2,t_2,V\right) \tag{9}$$

Note that there are two fundamental type terms in the force given in equation (6). The first term is radial and is proportional to **R** .The second term is non-radial and proportional to (β·**R**){**R**x(**R**xβ)}. The v^4/c^4 part of the first term will lead to Newton's universal law of gravitation. The v^4/c^4 part of the second term will lead to a new (previously unknown?) gravitational force term which causes a corkscrew type motion. Since the first term can be used to identify with Newton's universal force of gravitation, it will be calculated first.

Computation of Radial Force Term. The four radial force terms of equation (7) are given below where the coordinates are given explicitly:

$$\vec{F}_{2+,1+}=\frac{q_1q_2}{4\pi\varepsilon_0\left|\vec{r}_2-\vec{r}_1\right|^2}\hat{r}_{12}\left[1-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)^2-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)\cos^2\theta-\frac{1}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4-\frac{1}{4}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^2\theta+\frac{3}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^4\theta+...\right] \tag{10}$$

$$\vec{F}_{2-,1+}=\frac{q_1q_2}{4\pi\varepsilon_0\left|\vec{r}_2+\vec{A}_2-\vec{r}_1\right|^2}\hat{r}_{12}\left[1-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)^2-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)\cos^2\theta-\frac{1}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4-\frac{1}{4}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^2\theta+\frac{3}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^4\theta+...\right]$$

$$\vec{F}_{2+,1-}=\frac{q_1q_2}{4\pi\varepsilon_0\left|\vec{r}_2-\vec{r}_1-\vec{A}_1\right|^2}\hat{r}_{12}\left[1-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)^2-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)\cos^2\theta-\frac{1}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4-\frac{1}{4}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^2\theta+\frac{3}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^4\theta+...\right]$$

$$\vec{F}_{2-,1-}=\frac{q_1q_2}{4\pi\varepsilon_0\left|\vec{r}_2+\vec{A}_2-\vec{r}_1-\vec{A}_1\right|^2}\hat{r}_{12}\left[1-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)^2-\frac{1}{2}\left(\vec{\beta}_2-\vec{\beta}_1\right)\cos^2\theta-\frac{1}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4-\frac{1}{4}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^2\theta+\frac{3}{8}\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\cos^4\theta+...\right]$$

Now the force of gravity is normally measured in lab experiments where $r_2 - r_1 >> A_1$ and $r_2 - r_1 >> A_2$. Thus to good approximation the A_1 and A_2 terms in the denominator may be dropped such that

$$\vec{F}_{q2q1} = \frac{q_1 q_2}{4\pi\varepsilon_0 \left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}\left[1 - \frac{1}{2}\left(\vec{\beta}_2 - \vec{\beta}_1\right)^2 - \frac{1}{2}\left(\vec{\beta}_2 - \vec{\beta}_1\right)\cos^2\theta - \frac{1}{8}\left(\vec{\beta}_2 - \vec{\beta}_1\right)^4 - \frac{1}{4}\left(\vec{\beta}_2 - \vec{\beta}_1\right)^4\cos^2\theta + \frac{3}{8}\left(\vec{\beta}_2 - \vec{\beta}_1\right)^4\cos^4\theta + \ldots\right] \tag{11}$$

For the velocity terms in the [ ] of the expression for the force above, $\beta_2 \approx \beta_1$ and β_2 - $\beta_2 \approx 0$ for most laboratory measurements of gravity. In this case only the $A_1\omega_1$ and $A_2\omega_2$ terms are left as shown below where $q_1 = q_2 = e$ is the charge of the proton and $-e$ is the charge of the electron.

$$\begin{aligned}
\vec{F}_{2+,1+} &= \frac{\mathrm{e}^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}[1] \\
\vec{F}_{2+,1-} &= \frac{-\mathrm{e}^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}\left[1 - \left(\frac{A_1\omega_1}{c}\sin(\omega_1 t + \varphi_1)\right)^2\left(\frac{1+\cos^2\theta}{2}\right) + \left(\frac{A_1\omega_1}{c}\sin(\omega_1 t + \varphi_1)\right)^4\left(-\frac{1}{8} - \frac{1}{4}\cos^2\theta + \frac{3}{8}\cos^4\theta\right)\right] \\
\vec{F}_{2-,1+} &= \frac{-\mathrm{e}^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}\left[1 - \left(\frac{A_2\omega_2}{c}\sin(\omega_2 t + \varphi_2)\right)^2\left(\frac{1+\cos^2\theta}{2}\right) + \left(\frac{A_2\omega_2}{c}\sin(\omega_2 t + \varphi_2)\right)^4\left(-\frac{1}{8} - \frac{1}{4}\cos^2\theta + \frac{3}{8}\cos^4\theta\right)\right] \\
\vec{F}_{2-,1+} &= \frac{\mathrm{e}^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}\left[1 - \left(\frac{A_2\omega_2}{c}\sin(\omega_2 t + \varphi_2) + \frac{A_1\omega_1}{c}\sin(\omega_1 t + \varphi_1)\right)^2\left(\frac{1+\cos^2\theta}{2}\right) + \left(\frac{A_2\omega_2}{c}\sin(\omega_2 t + \varphi_2) + \frac{A_1\omega_1}{c}\sin(\omega_1 t + \varphi_1)\right)^4\left(-\frac{1}{8} - \frac{1}{4}\cos^2\theta + \frac{3}{8}\cos^4\theta\right)\right]
\end{aligned} \tag{12}$$

One can see that the sum of the first terms, i.e. 1 term in the [ ] of the four forces is just 0. The sum of the second terms in the [ ] does not sum to 0, since the cross term remains. Two parts of the third terms in the [ ] cancel, leaving the cross terms. The resulting sum of the four forces gives

$$\begin{aligned}
\vec{F} &= \vec{F}_{2+,1+} + \vec{F}_{2+,1-} + \vec{F}_{2-,1+} + \vec{F}_{2-,1-} \\
&= \frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}^2\, 2\frac{A_2\omega_2}{c}\sin\left(\omega_2 t_2 + \varphi_2\right)\frac{A_1\omega_1}{c}\sin\left(\omega_1 t_1 + \varphi_1\right)\left(\frac{1+\cos^2\theta}{2}\right) \\
&+ \frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}^4\, 4\frac{A_2\omega_2^3}{c}\sin^3\left(\omega_2 t_2 + \varphi_2\right)\frac{A_1\omega_1}{c}\sin\left(\omega_1 t_1 + \varphi_1\right)\left(-\frac{1}{8} - \frac{1}{4}\cos^2\theta + \frac{3}{8}\cos^4\theta\right) \\
&+ \frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}^4\, 4\frac{A_1\omega_1^3}{c}\sin^3\left(\omega_1 t_1 + \varphi_1\right)\frac{A_2\omega_2}{c}\sin\left(\omega_2 t_2 + \varphi_2\right)\left(-\frac{1}{8} - \frac{1}{4}\cos^2\theta + \frac{3}{8}\cos^4\theta\right) \\
&+ \frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2 - \vec{r}_1\right|^2}\hat{r}_{12}^4\, 6\frac{A_2^3\omega_2^3}{c}\sin^2\left(\omega_2 t_2 + \varphi_2\right)\frac{A_1\omega_1}{c}\sin\left(\omega_1 t_1 + \varphi_1\right)\left(-\frac{1}{8} - \frac{1}{4}\cos^2\theta + \frac{3}{8}\cos^4\theta\right)
\end{aligned} \tag{13}$$

Now the integrals in equation (9) can be evaluated. The symmetry of the integrals over φ_1 and φ_2 will cause the odd powers of $\sin(\omega_1 t_1 + \varphi_1)$ and $\sin(\omega_2 t_2 + \varphi_2)$ to average to zero, i.e.

$$\begin{aligned}
\frac{1}{T}\int_0^T \frac{1}{2\pi}\int_0^{2\pi}\sin\left(\omega t + \varphi\right)d\varphi\, dt &= \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\sin\left(\omega t + \varphi\right)d\varphi\, dt \\
&= \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2\pi}\int_0^{2\pi}\sin\left(x + \varphi\right)d\varphi\, dx \quad \left(x = \omega t\right) \\
&= \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2\pi}\left(\sin x\cos\varphi - \cos x\sin\varphi\right)d\varphi\, dx \\
&= \left(\frac{1}{2\pi}\right)^2\int_0^{2\pi}\left(-\cos x\cos\varphi + \sin x\sin\varphi\right)d\varphi\Big|_0^{2\pi} \\
&= \left(\frac{1}{2\pi}\right)^2\int_0^{2\pi}2\cos\varphi\, d\varphi = \left(\frac{1}{2\pi}\right)^2\left(2\sin\varphi\right)\Big|_0^{2\pi} \\
&= \left(\frac{1}{2\pi}\right)^2\left(-2\right)\left(0 - 0\right) = 0
\end{aligned} \tag{14}$$

Thus from the symmetry of the integrals the average force of equation (13) may be reduced to

$$\vec{F}(\vec{r})=\frac{1}{T_1}\int_0^{T_1}dt_1\,\frac{1}{T_2}\int_0^{T_2}dt_2\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi_1\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi_2\,\frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\,\vec{F}\left(r,\theta,\varphi,\vec{A}_1,\omega_1,\varphi_1,t_1,\vec{A}_2,\omega_2,\varphi_2,t_2\right) \tag{15}$$

$$\begin{aligned}
&=\frac{\omega_1}{2\pi}\int_0^{2\pi/\omega_1}dt_1\,\frac{\omega_2}{2\pi}\int_0^{2\pi/\omega_2}dt_2\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi_1\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi_2\\
&\quad\times\frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2-\vec{r}_1\right|^2}\,\hat{r}_{12}\,6\,\frac{A_2^2\omega_2^2}{c^2}\sin^2(\omega_2t_2+\varphi_2)\frac{A_1^2\omega_1^2}{c^2}\sin^2(\omega_1t_1+\varphi_1)\left(-\frac{1}{8}-\frac{1}{4}\cos^2\theta+\frac{3}{8}\cos^4\beta\theta\right)\\
&=-\frac{\omega_1}{2\pi}\int_0^{2\pi/\omega_1}dt_1\,\frac{\omega_2}{2\pi}\int_0^{2\pi/\omega_2}dt_2\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi_1\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi_2\,\frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2-\vec{r}_1\right|^2}\,\hat{r}_{12}\,6\,\frac{A_2^2\omega_2^2}{c^2}\sin^2(\omega_2t_2+\varphi_2)\frac{A_1^2\omega_1^2}{c^2}\sin^2(\omega_1t_1+\varphi_1)\frac{4}{15\pi}\\
&=-\frac{\omega_1}{2\pi}\int_0^{2\pi/\omega_1}dt_1\,\frac{\omega_2}{2\pi}\int_0^{2\pi/\omega_2}dt_2\,\frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2-\vec{r}_1\right|^2}\,\hat{r}_{12}\,6\,\frac{A_2^2\omega_2^2}{c^2}\,\frac{A_1^2\omega_1^2}{c^2}\,\frac{1}{2}\,\frac{4}{15\pi}\\
&=-\frac{e^2}{4\pi\varepsilon_o\left|\vec{r}_2-\vec{r}_1\right|^2}\,\hat{r}_{12}\,\frac{A_2^2\omega_2^2}{c^2}\,\frac{A_1^2\omega_1^2}{c^2}\,\frac{1}{2}\,\frac{2}{5\pi}\quad\text{(Attractive Forces Only)}
\end{aligned}$$

This attractive force is to be compared to Newton’s Universal Law of Gravitation.

$$\vec{F}(\vec{r})=-G\,\frac{m_1\,m_2}{4\pi\varepsilon_o\left|\vec{r}_2-\vec{r}_1\right|^2}\,\hat{r}_{12}\qquad\text{(Attractive Forces Only)}$$

where in the calculation of equation (15) above $-\sin\,\theta\,d\theta = d\cos\,\theta$, $x = \omega t$, $dx = \omega dt$, and

$$\begin{aligned}
\frac{1}{\pi}\int_0^{2\pi}\sin^2(\omega t+\varphi)d\varphi&=\frac{1}{2\pi}\int_0^{2\pi}\left(\sin^2\omega t\,\cos^2\varphi+2\cos\omega t\,\sin\omega t\,\sin\varphi\,\cos\varphi+\cos^2\omega t\,\sin^2\varphi\right)d\varphi\\
&=\left(\frac{1}{2\pi}\right)\left(\pi\cos^2\omega t+0+\pi\sin^2\omega t\right)=\frac{1}{2}\left(\cos^2\omega t+\sin^2\omega t\right)=\frac{1}{2}
\end{aligned}\tag{16}$$

and

$$\begin{aligned}
\frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\left(-\frac{1}{8}-\frac{1}{4}\cos^2\theta+\frac{3}{8}\cos^4\theta\right)&=\frac{1}{\pi}\int_{-1}^{+1}d\cos\theta\left(-\frac{1}{8}-\frac{1}{4}\cos^2\theta+\frac{3}{8}\cos^4\theta\right)\\
&=\frac{1}{\pi}\left(-\frac{1}{8}\cos\theta-\frac{1}{4}\frac{\cos^3\theta}{3}+\frac{3}{8}\frac{\cos^5\theta}{5}\right)\Big|_{-1}^{+1}\\
&=\frac{1}{\pi}\left(-\frac{1}{8}2-\frac{1}{4}\frac{2}{3}+\frac{3}{8}\frac{2}{5}\right)=-\frac{4}{15\pi}
\end{aligned}\tag{17}$$

Thus we need to see if the following relationship is reasonable.

$$G\,m_{g1}\,m_{g2}=\frac{1}{4\pi\varepsilon_o}\,\frac{2e^2}{5\pi}\,\frac{A_1^2\,\omega_1^2}{c^2}\,\frac{A_2^2\,\omega_2^2}{c^2}\tag{18}$$

Note that for two bodies with N_1 and N_2 atoms of atomic number Z_1 and Z_2 this formula becomes

$$G\, m_{g1}\, m_{g2} = \frac{1}{4\pi\varepsilon_o}\,\frac{2}{5\pi}\left(\frac{N_1 Z_1\, e\, A_1^2\, \omega_1^2}{c^2}\right)\left(\frac{N_2 Z_2\, e\, A_2^2\, \omega_2^2}{c^2}\right) \tag{19}$$

There will be a range of combinations of amplitude A and frequency ω for which equation (18) above holds. Although equation (15) looks very similar to Newton's Universal Law of Gravitation, it is very different. First it is a local contact force. Second it says gravity is decaying over time. Third the second term of the gravitational force will turn out to be a non-radial term causing many effects including the quantization of gravity. All of these gravitational effects must be observed in order to claim that this force law is valid. If the properties of the derived force of gravity are supported by experimental data, then it can be claimed to be superior to the previous theories of gravity such as Newton's Universal Law of Gravitation and Einstein's General Relativity Theory.

Acknowledgments. The author would like to acknowledge the world-class work of Dr. Andre Assis [10] on electrodynamics and gravity as well as his continual work on the electrodynamics of Weber. The derived universal electrodynamic force law approaches the empirical Weber force law in the limit $v/c \ll 1$ which was all that Weber could observe experimentally.

References.

1. de Coulomb, Charles Augustin (1785). Second Memoire sur l'Electricite et le Magnetism, *Histoire de l'Academie Royale de Sciences*, *pp*. 578-611.

2. Newton, Isaac (1687). *Principia.*

3. Poincaré, H. (1905), *C. R. Acad. Sci., 140*, 504, (1905) and also in (1906) *Rend. Pal., 21*, 129.

4. Einstein, Albert (1915), "Die Feldgleichungen der Gravitation", *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*: 844–847.

5. Einstein, Albert (1907). "Uber das Relativitatsprinzip und die aus demselben gezogenen Folgerungen". Jahrbuch der Radioactivitat und Electronik 4: 411-462.

6. Lucas, Charles W. Jr. and Joseph C. Lucas (2001). Weber's Force Law for Realistic Finite-Size Elastic Particles. *Journal of New Energy*, ***5(3)***, 70-89.

7. Lucas, Charles W. Jr. and Joseph C. Lucas (2003). Weber's Force Law for Finite-Size Elastic Particles. *Galilean Electrodynamics, 14*, 3-10.

8. Jackson, J. D. (1975). *Classical Electrodynamics-Second Edition* New York, NY: John Wiley and Sons, Inc.

9. Jackson, J. D. (1999). *Classical Electrodynamics-Third Edition* New York, NY: John Wiley and Sons, Inc.

10. Assis, A. K. T. (1995). Gravitation as a Fourth Order Electromagnetic Effect

in *Advanced Electromagnetism—Foundations, Theory and Applications*, Singapore: World Scientific, 314-331; Assis, A. K. T. (1989). On Mach's Principle. *Foundations of Physics Letters, 2*, 301-318; Assis, A. K. T. (1992). On the Absorption of Gravity. *Apeiron, 13*, 3-11; Assis, A. K. T. (1993). Compliance of a Weber Force Law for Gravitation with Mach's Principle. *Space and Time Problems in Modern Natural Science Part II* edited by P.N. Kropotkin et al. Saint Petersburg, 263-270; Assis, A. K. T. (1993). A Steady-State Cosmology in *Progress in New Cosmologies: Beyond the Big Bang* edited by H. C. Arp, C. R. Keys, and K. Rudnicki. New York, NY: Plenum Press 153-167.