

# Derivation of the Universal Force Law—Part 4

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**Abstract.** A new universal electromagnetic force law for real finite-size elastic charged particles is derived by solving simultaneously the fundamental empirical laws of classical electrodynamics, *i.e.* Gauss's laws, Ampere's generalized law, Faraday's law, and Lenz's law assuming Galilean invariance. This derived version of the electromagnetic force law incorporates the effects of the self-fields of real finite-size elastic particles as observed in particle scattering experiments. It can account for gravity, inertia, and relativistic effects including radiation and radiation reaction. The non-radial terms of the force law explain the experimentally observed curling of plasma currents, the tilting of the orbits of the planets with respect to the equatorial plane of the sun, and certain inertial gyroscope motions. The derived force law satisfies Newton's third law, conservation of energy and momentum, conservation of charge, and Mach's Principle. The mathematical properties of equations for the fundamental empirical laws and also Hooper's experiments [13] showing that the fields of a moving charge move with the charge require that the electrodynamic force be a contact force based on field extensions of the charge instead of action-at-a-distance. The Lorentz force is derived from Galilean invariance. The most general form of the force law, derived using all the higher order terms of the Galilean transformation, is assumed to be exact for all phenomena on all size scales. Arguments are given that this force law is superior to all previous force laws, *i.e.* relativistic quantum electrodynamics, gravitational, inertial, strong interaction and weak interaction force laws.

**Comparison of Derived Universal Force with Other Work.** In part 3 of this paper a universal electrodynamic force law for elastic finite-size charged particles was derived by solving, by substitution, a complete set of the fundamental empirical laws of classical electrodynamics, *i.e.* Gauss's laws, Ampere's law, Faraday's law, and Lenz's law assuming Galilean invariance. The mathematics of the derivation and Hooper's experimental results require that the electrodynamic force be a local contact force based on the electromagnetic fields being an extension of the charge. The Lorentz force law was derived from Galilean invariance implying that it is not a fundamental law of electrodynamics as previously thought in relativistic circles.

This approach dealt with a number of loose ends in electrodynamics. It derived the Lorentz force from Galilean invariance. It dealt with real elastic particles with finite size and self fields. It did not use fictitious action-at-a-distance forces between point particles, but used real contact forces based on the physical extension of charges via the charge's fields. It satisfied Mach's principle, which Einstein was never able to do with special relativity, by using Lenz's law. It used

the Galilean transformation based upon causality. It conserved energy and momentum 100% of the time compared to quantum electrodynamics which depends on the uncertainty principle to allow it to escape these conservation laws for brief periods of time. It only depended on the relative distance  $R$ , the relative velocity  $V$ , the relative acceleration  $A$ , and the relative  $dA/dt$  between the interacting charges, implying that the force has the same functional form for all observers irrespective of their states of motion or of their inertial frame of reference. It is a proper relativistic theory according to the classical definition, because it only depends on relative coordinates.

In part 3 of this paper the universal force law for constant velocity was derived. Then this result for the force was extended to cover acceleration terms by assuming that the force was continuous and well-behaved to obtain the result below.

$$\vec{F}(\vec{R}, \vec{V}, \vec{A}, \dots) = qq' \frac{(1 - \beta^2) \vec{R} + \frac{2\vec{R}^2}{c^2} \vec{A}}{\vec{R}^2 \left[ \vec{R}^2 - \frac{\{\vec{R} \times (\vec{R} \times \vec{\beta})\}^2}{\vec{R}^2} \right]^{1/2}} \quad (48)$$

$$- qq' \left( \frac{(\vec{\beta} \cdot \vec{R}) \vec{R} \times (\vec{R} \times \vec{\beta}) + (\vec{R} \cdot \vec{R}) \vec{R} \times \left( \vec{R} \times \frac{\vec{A}}{c^2} \right)}{(1 - \beta^2) \left[ \vec{R}^2 - \frac{\{\vec{R} \times (\vec{R} \times \vec{\beta})\}^2}{\vec{R}^2} \right]^{3/2}} \right)$$

This force, in the limit of constant velocity with acceleration  $A = 0$ , corresponds to the covariant relativistic electrodynamic force of equation (12 in part 1) based on Maxwell's equations [9b p. 555 or 9c p. 560]. It appears to be fully "relativistic" without any reference to Einstein's Special Relativity Theory or retarded fields or their unique assumptions. It accounts for relativistic radiation effects as will be shown in this part of the paper.

There are other versions of the electrodynamic force in the scientific literature. This derived version of the electromagnetic force law is equivalent to Phipps's [4] and Wesley's [5] proposed relativistic version in the limiting case of transverse or circular motion with  $A = 0$  to order  $V^2/c^2$ .

$$\vec{F}(\vec{R}, \vec{V}, \vec{A})_{\vec{R} \perp \vec{V}, \vec{A}=0} = \frac{qq'\vec{R}}{R^3} \left[ \frac{(1 - \beta^2)}{(1 - \beta^2 \sin^2 \theta)^{1/2}} \right]_{\theta=\pi/2} = \frac{qq'\vec{R}}{R^3} \left[ 1 - \frac{\vec{v}^2}{c^2} \right]^{1/2} \approx \vec{F}_{Phipps} \quad (49)$$

$$= \frac{qq'\vec{R}}{R^3} \left[ 1 - \frac{\vec{v}^2}{c^2} \right]^{1/2} \approx \vec{F}_{Wesley}$$

It is also equivalent to Weber's [6] version of the force law for linear motion, *i.e.*  $\vec{V} \parallel \vec{R} \parallel \vec{A}$  such that  $\sin \theta = 0$ .

$$\vec{F}(\vec{R}, \vec{V}, \vec{A})_{\vec{R} \parallel \vec{V} \parallel \vec{A}} = \frac{qq'}{R^3} \left[ \left( 1 - \frac{\vec{V}^2}{c^2} \right) \vec{R} + \frac{2\vec{R}^2}{c^2} \vec{A} \dots \right] \approx \vec{F}_{Weber} \quad (50)$$

In the non-relativistic limit  $v \ll c$  the magnetic field which is proportional to  $\vec{V} \times \vec{E}$  may be neglected such that

$$\begin{aligned} \vec{F}_{V \ll c} &\approx q\vec{E}_{V \ll c} \approx q\vec{E}_o + q(\vec{E}_a + \vec{E}_{Rad}) \approx \frac{qq'\vec{R}}{R^3} + \frac{qq'}{c^2 R} [2\vec{A} + \hat{R}(\hat{R} \cdot \vec{A}) - \vec{A}] \\ &= \frac{qq'\vec{R}}{R^3} + \frac{qq'}{c^2 R} [\vec{A} + \hat{R}(\hat{R} \cdot \vec{A})] \end{aligned} \quad (51)$$

This is identical to the result for the Taylor series expansion of the Lienard-Wiechert electric field for  $\mathbf{V} = 0$  and  $\mathbf{A} > 0$  [9c, p. 698 problem 14.2].

**Acceleration Fields and Radiation.** In order to determine the radiation fields due to acceleration, it is convenient to define the electromagnetic fields and force  $\mathbf{F}$  in terms of their electric and magnetic components.

$$\begin{aligned} \vec{F}_E &= q\vec{E}(\vec{R}) = q[\vec{E}_o(\vec{R}) + \vec{E}_i(\vec{R})] \\ \vec{F}_M &= \frac{\vec{V}}{c} \times [\vec{B}_o(\vec{R}) + \vec{B}_i(\vec{R})] \end{aligned} \quad (52)$$

The induced fields may be defined in terms of the type of motion that produced them, *i.e.*

$$\begin{aligned} \vec{E}_i(\vec{R}) &= \vec{E}_V(\vec{R}) + \vec{E}_A(\vec{R}) + \vec{E}_{Rad}(\vec{R}) + \dots \\ \vec{B}_i(\vec{R}) &= \vec{B}_V(\vec{R}) + \vec{B}_A(\vec{R}) + \vec{B}_{Rad}(\vec{R}) + \dots \end{aligned} \quad (53)$$

Thus

$$\vec{F} = \vec{F}_{Eo} + \vec{F}_{Ev} + \vec{F}_{Ea} + \dots + \vec{F}_{Bo} + \vec{F}_{Bv} + \vec{F}_{Ba} + \dots + \vec{F}_{Rad} \quad (54)$$

Since the velocity term in the induced fields is known experimentally to *not* obey the superposition principle, it is assumed that the acceleration and higher order time derivative terms also do not obey the superposition principle. Thus each of these fields should have unique properties and may correspond to the unique static, velocity and acceleration electric and magnetic fields reported by Hooper [13, pp. 9-15].

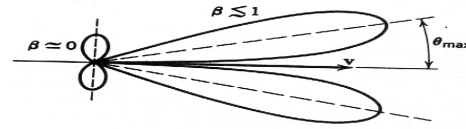
Comparing equation (54) with equation (48) one obtains

$$\begin{aligned}\bar{F}_{Eo} + \bar{F}_{Ev} &= \frac{qq'\bar{R}}{R^3} \frac{(1-\beta^2)}{(1-\beta^2 \sin^2 \theta)^{3/2}} = q'(\bar{E}_o + \bar{E}_v) \\ \bar{F}_{Bo} + \bar{F}_{Bv} &= \frac{qq'\bar{R}}{R^3} \frac{(1-\beta^2)\beta^2 \sin^2 \theta}{(1-\beta^2 \sin^2 \theta)^{3/2}} + \frac{qq'(1-\beta^2)(\vec{\beta} \cdot \vec{R})\vec{R} \times (\vec{R} \times \vec{\beta})}{R^4 (1-\beta^2 \sin^2 \theta)^{3/2}} \\ \bar{F}_{Rad} &= \frac{qq'}{c^2 R^3} (1-\beta^2) \frac{\vec{R} \times (\vec{R} \times \vec{A})}{(1-\beta^2 \sin^2 \theta)^{3/2}} = q\bar{E}_{Rad}\end{aligned}\quad (55)$$

where the explicit vector dependence has been removed from the denominator.

### Radiation from Real Finite-Size Particles.

If a charged particle is accelerated, it is observed to give off radiation. This radiation is due to the curling motion of the electric and magnetic fields produced by the acceleration. The instantaneous energy flux of the radiation is given by the Poynting vector  $\mathbf{S}$  [9c, p. 665].



**Figure 6.**

Searchlight Effect Radiation Pattern for Charge Accelerated in its Direction of Motion

$$\bar{S} = \frac{c}{4\pi} \bar{E}_{Rad} \times \bar{B}_{Rad} = \frac{c}{4\pi} |\bar{E}_{Rad}|^2 \hat{R} = \frac{q^2}{4\pi c^3 R^2} \frac{(1-\beta^2)^2}{(1-\beta^2 \sin^2 \theta)^3} |\hat{R} \times (\hat{R} \times \vec{A})|^2 \quad (56)$$

where  $\theta$  is the angle between  $\mathbf{R}$  and  $\mathbf{V}$ , and all observed radiation is due to acceleration. Thus the power radiated per solid angle is

$$\frac{dP}{d\Omega} = R^2 |\bar{S}| = \frac{q^2}{4\pi c^3} \frac{(1-\beta^2)^2}{(1-\beta^2 \sin^2 \theta)^3} |\hat{R} \times (\hat{R} \times \vec{A})|^2 \quad (57)$$

In the non-relativistic limit  $V/c \ll 1$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\vec{A}|^2 \sin^2 \theta' \quad (58)$$

where  $\theta$  is the angle between  $\mathbf{R}$  and  $\mathbf{A}$ . The total instantaneous power radiated is obtained by integrating  $dP/d\Omega$  over all solid angles.

This is the standard Larmor formula for total power radiated from a non-relativistic accelerated charge [9a p. 469 or 9b p. 659 or 9c, p. 665]. For the relativistic case where  $\mathbf{V}$  is perpendicular to  $\mathbf{R}$  such that  $\sin \theta = 1$  or  $\mathbf{R} \cdot \mathbf{V} = 0$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \frac{(1-\beta^2)^2}{(1-\beta^2 \sin^2 \theta)^3} |\bar{A}|^2 \sin^2 \theta$$

$$P = \frac{2q^2}{3c^3} \gamma^4 |\bar{A}|^2 (1-\beta^2) = \frac{2q^2}{3c^3} \gamma^4 \left[ \bar{A}^2 - (\bar{V} \times \bar{A})^2 \right] \quad (60)$$

where  $\bar{A}^2(1-\beta^2) = \bar{A}^2 - (\bar{V}/c \times \bar{A})^2$  since  $\bar{V} \cdot \bar{A} = 0$ . This is the fully relativistic Lienard result first obtained in 1898 [9a p. 470, 9b p. 660, 9c p. 666]. Note that there is a relationship between  $\theta$  and  $\theta'$ , *i.e.*  $\sin \theta \rightarrow 1$  as  $\sin \theta' \rightarrow 0$ . This causes all the radiation to be emitted at very small angles  $\theta$  producing the search light effect observed in particle accelerators and shown in Figure 6.

**Nature of Light and its Circuit.** The electrodynamic force is a local contact type of force in which the fields are an extension of the charge. The fields have tensile strength and extend to great distances. In this case light is a ripple of angular momentum in the fields of the particle. All field lines start on a charge and end on a charge as shown in Figures 2b and 2c in Part 1. Since most charges appear in neutral pairs throughout the universe, most of the field lines return to the nearby charge of the opposite sign. Thus there is a circuit of some sort that brings light back to its original source, since it travels on the fields of the particle that emitted it. This notion is similar to some ideas in General Relativity Theory that matter can bend the direction of light. These field extensions of the charge define the fabric of space. They physically define the ether of General Relativity theory which Einstein was never able to do.

**Summary of Results.** A universal electrodynamic force law for elastic finite-size charged particles was derived by simultaneously solving, by substitution, the fundamental empirical laws of classical electrodynamics, *i.e.* Gauss's laws, Ampere's law, Faraday's law, Lorentz's law, and Lenz's law assuming Galilean invariance. The mathematics of the derivation and Hooper's experimental results [13] require that the electrodynamic force be a local contact force based on the electromagnetic fields being an extension of the charges.

This newly derived version of Weber's force law appears to be fully "relativistic" without any reference to Einstein's Special Relativity Theory or retarded fields. It accounts for relativistic effects including radiation. Finally the field extensions of the charges in the universe physically define the fabric of space or the ether of General Relativity theory. Einstein was never able to physically define the fabric of space. He was only able to describe it mathematically.

This approach deals with a number of loose ends in electrodynamics. It derives the Lorentz force from Galilean invariance. It deals with real elastic particles with finite size and self fields. It does not use fictitious action-at-a-distance forces but uses real contact forces based on the physical extension of charges via the charge's fields. It satisfies Mach's principle, which Einstein was never able to do

with special relativity, by using Lenz's law. It uses the Galilean transformation based upon causality. It conserves energy and momentum 100% of the time compared to quantum electrodynamics which depends on the uncertainty principle to allow it to escape these conservation laws for brief periods of time. It only depends on the relative distance  $R$ , the relative velocity  $V$  the relative acceleration  $A$  and the relative  $dA/dt$  between the interacting charges implying that the force has the same value for all observers irrespective of their states of motion or of their inertial frame of reference. It is a proper relativistic theory according to the classical definition, because it only depends on relative coordinates.

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