Derivation of the Universal Force Law—Part 3

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Abstract. A new universal electromagnetic force law for real finite-size elastic charged particles is derived by solving simultaneously the fundamental empirical laws of classical electrodynamics, *i.e.* Gauss's laws, Ampere's generalized law, Faraday's law, and Lenz's law assuming Galilean invariance. This derived version of the electromagnetic force law incorporates the effects of the self-fields of real finite-size elastic particles as observed in particle scattering experiments. It can account for gravity, inertia, and relativistic effects including radiation and radiation reaction. The non-radial terms of the force law explain the experimentally observed curling of plasma currents, the tilting of the orbits of the planets with respect to the equatorial plane of the sun, and certain inertial gyroscope motions. The derived force law satisfies Newton's third law, conservation of energy and momentum, conservation of charge, and Mach's Principle. The mathematical properties of equations for the fundamental empirical laws and also Hooper's experiments showing that the fields of a moving charge move with the charge require that the electrodynamic force be a contact force based on field extensions of the charge instead of action-at-a-distance. The Lorentz force is derived from Galilean invariance. The most general form of the force law, derived using all the higher order terms of the Galilean transformation, is assumed to be exact for all phenomena on all size scales. Arguments are given that this force law is superior to all previous force laws, *i.e.* relativistic quantum electrodynamics, gravitational, inertial, strong interaction and weak interaction force laws.

Case for Constant Relative Velocity. In the part 2 of this paper was a derivation of Ampere's force law between elements of current loops and the generalized version of Ampere's force law and the proof that both satisfy Newton's third law. Also included was a derivation of the Biot-Savart law and the Grassman force law from the generalized Ampere force law. The derivation showed that both the Biot-Savart law and the Grassman force law satisfy Newton's third law when one notes that the additional term in the generalized Ampere force law, that is missing in the Grassman force law, gives no contribution for closed current loops. *But remember, Ampere's force law only applies to closed current loops!!*

Using the Grassman form of the generalized Ampere force law of equation (A19) for a single point element of charge q moving with a relative velocity $\mathbf{v}$, the induced flux density $\vec{B}_i(\vec{r},t)$ will be

$$\vec{B}_i(\vec{r},t) = q\frac{\vec{v}\times\vec{r}\,'}{c\left|\vec{r}\,'\right|^3} = \frac{\vec{v}}{c}\times\vec{E}_0(\vec{r}\,',t') \tag{15}$$

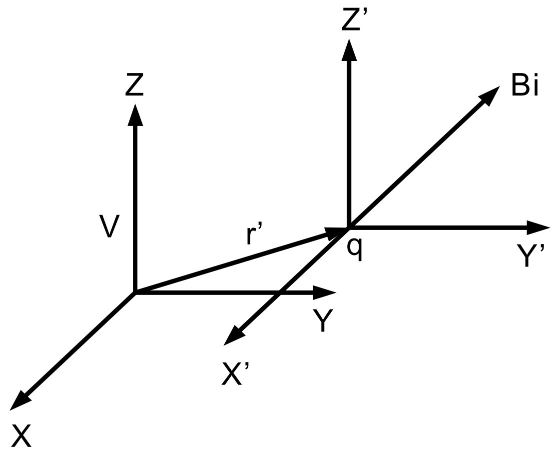


Figure 1.

Induction Field Bi in the primed reference frame as observed in the unprimed reference frame due to charge q moving with velocity $\vec{v}$.

where the more familiar relativistic type notation of Figure 1 has been used for clarity. Note that equation (15) for a point element of charge gives the transformation of the $\vec{E}_o(\vec{r}',t')$ field in the moving frame of reference to the induced field $\vec{B}_i(\vec{r},t)$ in the observer's frame of reference. This is assumed valid for all velocities $\vec{v}$ whether constant in time or changing, and will be used to obtain the fields of a charged particle with internal charge distribution. Note that if Ampere's law, as represented by equation (15), is cast into its usual Maxwell equation form

$$\vec{\nabla} \times \vec{B}(\vec{r},t) = \frac{4\pi}{c} \vec{J}(\vec{r},t) \qquad (16)$$

the reference frame transformation information of equation (15) is lost. Jackson [9a *p.* 138 or 9b *p.* 174 or 9c *p.* 179] points out that

$$\nabla \times \vec{B}(\vec{r},t) = \frac{4\pi}{c} \vec{J}(\vec{r},t) + \frac{1}{c} \nabla \int \frac{\nabla' \cdot \vec{J}(\vec{r}',t')}{|\vec{r}-\vec{r}'|} d^3 r' \qquad (17)$$

The second term on the right is not zero for finite size particles and induced self-field effects. For these reasons it appears that the covariant form of electrodynamics, based on Maxwell's equations, is technically incorrect. It is certainly not as fundamental as the empirical laws upon which Maxwell's equations are based.

From equation (15) the motion of the elementary charge q will produce an induced magnetic field $\vec{B}_i(\vec{r},t)$. In order to obtain the $\vec{B}_i(\vec{r},t)$ field that an observer would see in his reference frame, a coordinate substitution, known as the Galilean transformation, is used to obtain the expression for $\vec{B}_i(\vec{r},t)$ in terms of the unprimed coordinates, *i.e.*

$$\vec{r}' = \vec{r} - \vec{v}\,t - \frac{1}{2}\vec{a}\,t^2 - \frac{1}{6}\dot{\vec{a}}\,t^3 \ldots \quad \text{and} \quad t' = t \quad c' \neq c \qquad (18)$$

where **v** is assumed to be constant in time and **a** = 0. Note that classical electrodynamics was originally based on Galilean relativity which assumed instant action-at-a-distance for longitudinal forces due to the Coulomb gauge potential [9a *pp.* 181-183, 9b *pp.* 220-223, 9c *pp.* 240-242] and absolute time as defined above such that the retarded time $\tau = t' - t = 0$. Thus it was inconsistent in logic to introduce retardation effects in classical electrodynamics.

Note that $\vec{B}_i(\vec{r},t)$ will be time varying. According to Faraday's law [9a *pp.* 170-173 or 9b *pp.* 210-213 or 9c *pp.* 208-211] $d\vec{B}_i(\vec{r},t)/dt$ introduces an additional electric field $\vec{E}_i(\vec{r},t)$. The distribution of charge within the finite-size elastic elementary charged particle rearranges itself to be consistent with the induced fields. This rearrangement causes a change in the internal binding energy of the particle, and the kinetic energy is stored in the particle's external electromagnetic fields. Using the fundamental empirical equations of electrodynamics one can calculate the induced fields in order to obtain the total fields of the moving charged particle, *i.e.* where $\vec{E}_o(\vec{r},t')$ is the electrostatic field and $\vec{E}_i(\vec{r},t)$ is the induced field due to motion **v**.

$$\vec{E}(\vec{r},t) = \vec{E}_o(\vec{r},t) + \vec{E}_i(\vec{r},t) \qquad \vec{B}(\vec{r},t) = \vec{B}_o(\vec{r},t) + \vec{B}_i(\vec{r},t) \tag{19}$$

In 1957 Cullwick [12] published results of his research indicating that the magnetic flux loops discovered by Oersted [26] were actually in motion along the linear conductor in the direction of the electron current giving rise to it. The loops moved with the electron drift velocity. This result was confirmed experimentally by Hooper [13]. This means that when motion is involved $\vec{E}(\vec{r},t)$ must be written in terms of the coordinates of its rest frame, *i.e.* $\vec{E}(\vec{r},t)$ becomes $\vec{E}(\vec{r}',t) = \vec{E}(\vec{r} - \vec{v}t - 1/2\,at^2 - \ldots, t)$. Faraday believed that the fields were an extension of the charge to infinity, that the field lines had tensile strength to attract and repel as shown in Figures 2a, 2b and 2c, and that light was merely a ripple of angular momentum in the fields of the charge [29].

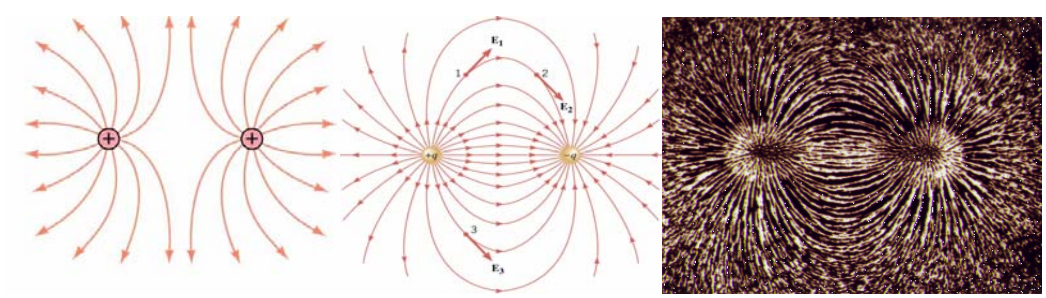


Figure 2a. Repulsion of Positive charges
Figure 2b. Attraction of Opposite Charges
Figure 2c. Iron Filings Showing Field Lines

Another significant aspect of this work is that the induced $\vec{B} \times \vec{v}$ field is not electrostatic in nature. According to Culwick [12], Hooper [13], and Moon and Spencer [14], this means that the superposition principle as applied to electrical fields does not hold for the $\vec{B}\times\vec{v}$ generated fields. Thus in electrodynamics one must explicitly keep track of both electrostatic fields and the induced fields. Using the basic equations of electrodynamics one must calculate explicitly the induced fields in order to obtain the total fields of the moving charged particle. (Note that the covariant form of electrodynamics based upon Maxwell's equations assumes that the superposition principle holds and does not treat electrostatic and induced fields separately in disagreement with the experimental results cited [13]).

From the discussion above, equation (15) for a charged particle with finite size and internal charge structure with total charge q must be written in terms of the position of the charge in its current rest frame (See Figure 1 and note origin of $\vec{B}_i$ in primed coordinate system).

$$\vec{B}\left(\vec{r}\,',t'\right)=\vec{B}_i\left(\vec{r}-\vec{v}t-1/2\,\vec{a}t^2-\ldots,t\right)=\frac{\vec{v}}{c}\times\left(\vec{E}_o\left(\vec{r}\,',t'\right)+\vec{E}_i\left(\vec{r}\,',t'\right)\right) \tag{20}$$

Note that $\vec{B}_i$ is not a function of $(\vec{r},t)$ but of $(\vec{r}\,',t')=\left(\vec{r}-\vec{v}\,t-1/2\,\vec{a}\,t^2-\ldots,t\right)$ due to the fact that the induced field remains attached to the charge and reflects all aspects of the motion of the charge. Assume that the particle with charge q is moving with relative velocity v in the z-direction and that from symmetry $\vec{E}_i\left(\vec{r}\,',t'\right)=E_i\left(\vec{r}\,',t'\right)\,\vec{r}\,'/\left|\vec{r}\,'\right|$ such that

$$\vec{B}_i\left(\vec{r}-\vec{v}t-1/2\,\vec{a}t^2-\ldots,t\right)=\frac{\vec{v}\times\hat{r}'}{c}\frac{q}{\left|\vec{r}\,'\right|^2}+\vec{v}\times\vec{E}_i\left(\vec{r}\,',t'\right) \tag{21}$$

Using the Galilean transformation of Equation (18) for constant relative velocity one obtains

$$\vec{B}_i\left(\vec{r}-\vec{v}t,t\right)=\frac{v}{c}r\sin\theta\frac{q}{\left|\vec{r}-\vec{v}t\right|^3}\hat{\varphi}'+\frac{v}{c}\sin\theta\,E_i\left(\vec{r}-\vec{v}t,t\right)\hat{\varphi}' \tag{22}$$

where spherical coordinates are used with

$$\frac{\partial\vec{B}_i\left(\vec{r}-\vec{v}\,t,t\right)}{c\,\partial t}=\frac{v}{c}r\sin\theta\,\hat{\varphi}'\left[\frac{3qv\left(r\cos\theta-vt\right)}{c\left|\vec{r}-\vec{v}\,t\right|^5}+\frac{\partial}{rc\partial t}E_i\left(\vec{r}-\vec{v}\,t,t\right)\right] \tag{23}$$

and

$$\begin{aligned}\left|\vec{r}\,'\right|&=\left|\vec{r}-\vec{v}\,t\right|=\sqrt{r^2\sin^2\theta+\left(r\cos\theta-vt\right)^2}\\ \vec{v}\times\vec{r}\,'&=\vec{v}\times\left(\vec{r}-\vec{v}\,t\right)=\vec{v}\times\vec{r}=vr\sin\theta\,\hat{\varphi}'\end{aligned} \tag{24}$$

Now the induced time varying magnetic field $\vec{B}_i\left(\vec{r}-\vec{v}t,t\right)$ causes an electric field $\vec{E}_i\left(\vec{r}-\vec{v}t,t\right)$ to be induced. According to Faraday's law [9a *p.* 173 or 9b *p.* 213 or 9c *p.* 211] the changing magnetic flux linked by a circuit is proportional to the induced electric field around the circuit, *i.e.*

$$\int_C\vec{E}_i'\left(\vec{r}\,',t'\right)\cdot d\vec{l}\,'=-\frac{1}{c}\frac{d}{dt}\int_S\vec{B}_i\left(\vec{r}-\vec{v}t\right)\cdot\hat{n}\,da' \tag{25}$$

where the circuits and fields are defined in Figure 3. Note that Faraday's law gives the relationship between $\vec{E}_i'(\vec{r}\,',t')$ in the moving frame to $\vec{B}_i(\vec{r}-\vec{v}t,t)$ in the observer's frame. This relationship is lost if one casts Faraday's law into its usual covariant Maxwell equation form as shown below.

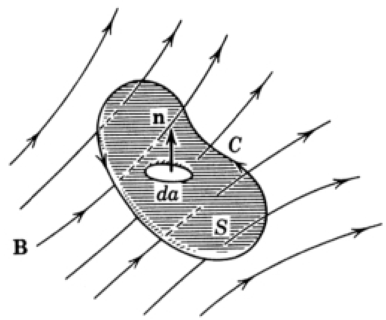


Figure 3.
Definition of Fields
for Faraday's law

Faraday's law can be used as a second equation to go with equation (20) to solve for the induced $\vec{B}_i$ and $\vec{E}_i$ fields. *Eventually a complete set of all the fundamental equations of electrodynamics will be required to obtain the solution.* Faraday's law can be rewritten using Stokes Theorem as

$$\int_C \left(\vec{E}_i'(\vec{r}\,',t') - \frac{1}{c}\left(\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t)\right)\right)\cdot dl' = \frac{1}{c}\oint_S \frac{\partial \vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t}\cdot\hat{n}\,da' \qquad (26a)$$

where use of the convective derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\nabla' \qquad (26b)$$

gives

$$\begin{aligned} \frac{d\vec{B}_i(\vec{r}-\vec{v}t,t)}{dt} &= \frac{\partial \vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}'\right)\vec{B}_i(\vec{r}-\vec{v}t,t) \\ &= \frac{\partial \vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t} + \vec{\nabla}'\times\left[\vec{B}_i(\vec{r}-\vec{v}t,t)\times\vec{v}\right] + \vec{v}\left(\vec{\nabla}'\cdot\vec{B}_i(\vec{r}-\vec{v}t,t)\right) \\ &= \frac{\partial \vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t} + \vec{\nabla}'\times\left[\vec{B}_i(\vec{r}-\vec{v}t,t)\times\vec{v}\right] \qquad (26c) \end{aligned}$$

since $\vec{\nabla}'\cdot\vec{B} = 0$.

This is a statement of Faraday's law applied to a moving circuit C. We could also apply Faraday's law to a circuit C instantaneously at rest with $v = 0$. In this case one obtains

$$\oint_C \left[\vec{E}_i'(\vec{r}-v't,t)\right]\cdot dl' = \frac{1}{c}\oint_S \frac{\partial B_i(\vec{r}-\vec{v}t,t)}{\partial t}\cdot\hat{n}'\,da' \qquad (27)$$

Galilean invariance requires that in this case $\vec{E}' = \vec{E}$ and $\vec{F}' = \vec{F}$. Thus the left-hand sides of equations (26a) and (27) are equal giving

$$\vec{E}_i'(\vec{r}\,',t') = E_i(\vec{r}-\vec{v}t,t) + \frac{1}{c}\left[\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t)\right] \tag{27a}$$

or

$$\vec{F}'(\vec{r}\,',t') = \vec{F}_o'(\vec{r}\,',t') + \vec{F}_i'(\vec{r}\,',t') = q\vec{E}_o'(\vec{r}\,',t') + q\vec{E}_1'(\vec{r}\,',t')$$

$$= q\vec{E}_o(\vec{r}-\vec{v}t,t) + q\vec{E}_i(\vec{r}-\vec{v}t,t) + \frac{q}{c}\left[\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t)\right]$$

or

$$\vec{F}'(\vec{r}\,',t') = q\vec{E}(\vec{r}-\vec{v}t,t) + \frac{q}{c}\left[\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t)\right] \qquad (\text{Lorentz's Force Law})$$

where static fields are identical in all frames of reference. Note that the Lorentz force law has been derived from Galilean invariance plus the experimental fact that fields are a physical extension of the charge making the electromagnetic force a contact type force.

Faraday's law of equation (26a) can be put into differential form by use of Stokes's theorem and equations (26b, 27a). The transformation of the line integral for the electric field into a surface integral leads to

$$\int_S \left(\vec{\nabla}'\times\vec{E}_i(\vec{r}-\vec{v}t,t) + \frac{1}{c}\frac{\partial}{\partial t}\vec{B}_i(\vec{r}-\vec{v}t,t)\right)\cdot\hat{n}'\,da' = 0 \tag{28}$$

Since the circuit C and the bounding surface S are arbitrary, the integrand must vanish at all points in space. Thus the differential form of Faraday's law for the special case $\vec{v}=0$ is

$$\vec{\nabla}'\times\vec{E}_i(\vec{r}-\vec{v}t,t) = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_i(\vec{r}-\vec{v}t,t) \tag{29}$$

Note that for self-consistency the charge distribution within the finite-size elementary particle has been altered to produce the field $\vec{E}(\vec{r}-\vec{v}t,t) = \vec{E}_o(\vec{r}-\vec{v}t,t) + \vec{E}_i(\vec{r}-\vec{v}t,t)$, because the charge distribution is assumed to be in equilibrium with all forces or fields. Using equation (23) one may write Faraday's law in spherical coordinates as

$$\begin{aligned}\vec{\nabla}'\times\vec{E}_i(\vec{r}-\vec{v}t,t) = {}& \frac{\hat{r}'}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\,\vec{E}_i(\vec{r}\,',t)\cdot\hat{\varphi}'\right) - \frac{\partial}{\partial\varphi}\left(\vec{E}_i(\vec{r}\,',t)\cdot\hat{\theta}'\right)\right] \\ & + \hat{\theta}'\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\vec{E}_i(\vec{r}\,',t)\cdot\hat{r}'\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(r\vec{E}_i(\vec{r}\,',t)\cdot\hat{\varphi}'\right)\right] \\ & + \frac{\hat{\varphi}'}{r}\left[\frac{\partial}{\partial r}\left(r\vec{E}_i\cdot\hat{\theta}'\right) - \frac{\partial}{\partial\theta}\left(\vec{E}_i(\vec{r}\,',t)\cdot\hat{r}'\right)\right] \\ = {}& \frac{1}{c}\frac{\partial}{\partial t}\vec{B}_i(\vec{r}\,',t) = -\frac{v}{c}r\sin\theta\left[\frac{3qv(r\cos\theta-vt)}{c\left|\vec{r}-\vec{v}t\right|^5} + \frac{1}{rc}\frac{\partial}{\partial t}E_i(\vec{r}-\vec{v}t,t)\right]\hat{\varphi}'\end{aligned} \tag{30}$$

The symmetry of $\partial\vec{B}_i(\vec{r}-v't)/\partial t \propto \hat{\varphi}'$ and $\vec{E}_i(\vec{r}-v't) \propto \hat{r}'$ allows equation (30) to be reduced to

$$-\frac{1}{r}\frac{\partial}{\partial\theta'}E_i\left(\vec{r}-\vec{v}t,t\right)\hat{\varphi}' = -\frac{v}{c}r\sin\theta\left[\frac{3qv\left(r\cos\theta - vt\right)}{c\left|\vec{r}-\vec{v}t\right|^5}+\frac{1}{rc}\frac{\partial}{\partial t}E_i\left(\vec{r}-\vec{v}t,t\right)\right]\hat{\varphi}' \quad (31)$$

and

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi'}E_i\left(\vec{r}-\vec{v}t,t\right)\hat{\theta}' = 0$$

or rearranging the top equation and integrating over θ obtain

$$\int_0^{\theta'} dE_i\left(\vec{r}-\vec{v}t,t\right) = E_i\left(\vec{r}-\vec{v}t,t\right) - E_i\left(\vec{r}-\vec{v}t,t\right)\Big|_{\theta'=0} \quad (32)$$

$$= \frac{3v^2r^2q}{c^2}\int_0^{\theta'}\frac{\left(r\cos\theta' - vt\right)}{\left|\vec{r}-\vec{v}t\right|^5}\sin\theta'\,d\theta'$$

$$+\frac{vr^2}{c}\int_0^{\theta'}\frac{\partial}{rc\,\partial t}E_i\left(\vec{r}-\vec{v}t,t\right)\sin\theta'\,d\theta'$$

where the constant $\vec{E}_i\left(\vec{r}-\vec{v}t,t\right)\Big|_{\theta=0}$ diminishes the original electrostatic field. From Lenz's law and symmetry of local forces, $\vec{E}_i\left(\vec{r}-\vec{v}t,t\right)\Big|_{\theta=0}$ should oppose the induced field $E_i\left(\vec{r}-\vec{v}t,t\right)\hat{r}'$, which is proportional to the moving static field $E_o\left(\vec{r}-\vec{v}t,t\right)\hat{r}'$, *i.e.*

$$E_i\left(\vec{r}-\vec{v}t,t\right)\Big|_{\theta'=0}\hat{r}' = -\lambda E_o\left(\vec{r}-\vec{v}t,t\right)\hat{r}' \quad (33)$$

The use of Lenz's law allows one to satisfy Mach's principle [28]. According to Mach any correct physical theory must take into account all the charges and masses in the universe in a consistent way, because the electromagnetic and gravitational forces have infinite range. Since all massive particles seem to have charge structure, the use of Lenz's law appears to satisfy Mach's principle by taking into account the electromagnetic interaction of all other particles in the universe. Einstein's Special Relativity Theory (SRT) incorporating the Lorentz transformation does not satisfy Mach's principle. The covariant form of electrodynamics based on Maxwell's equations also fails to satisfy Mach's principle.

Equation (32) can be solved by iterative substitution for $E_i\left(\vec{r}-\vec{v}t,t\right)$ where each successive iteration takes into account the next order of the induced field effect. In order to obtain the results observed in the laboratory frame, it is necessary to evaluate $E_i\left(\vec{r}-\vec{v}t,t\right)\Big|_{t=0}$ at t = 0 when the moving frame coincides with the laboratory frame after all terms for a given iteration have been evaluated. In order to simplify the iteration of successive terms the $\vec{r}\,' = \vec{r}-\vec{v}t$ terms are left in place in order to keep track of the correct power of $\vec{r}\,'$ for the derivative in the iterative term. The $vt\left(\cos\theta - 1\right)$ terms are explicitly dropped for t = 0. Substituting equation (33) into equation (32) and using $\beta = v/c$ obtain

$$E_i\left(\vec r-\vec v t,t\right)\Big|_{\,t=0}+\lambda E_o\left(\vec r-\vec v t,t\right)\Big|_{t=0} \tag{34}$$

$$=3r^2\beta^2\left[\frac{q}{\left|\vec r-\vec v t\right|^5}\left(\frac{r}{2}\sin^2\theta'+vt\left(\cos\theta'-1\right)\right)\right]_{t=0}$$

$$+\beta r^2\int_0^{\theta'}\frac{\partial}{rc\,\partial t}E_i\left(\vec r-\vec v t,t\right)\sin\theta'\,d\theta'\Big|_{t=0}$$

Iterating equation (34) obtain

$$E_i\left(\vec r-\vec v t,t\right)\Big|_{\,t=0}=\frac{3}{2}\beta^2r^3\frac{q}{\left|\vec r-\vec v t\right|^5}\sin^2\theta\Bigg|_{\,t=0}-\lambda r\frac{q}{\left|\vec r-\vec v t\right|^3}\Bigg|_{t=0} \tag{35}$$

$$+\beta r^2\int_0^{\theta}\frac{\partial}{rc\,\partial t}\left(\frac{3}{2}\beta^2r^3\frac{q}{\left|\vec r-\vec v t\right|^5}\sin^2\theta'-\lambda r\frac{q}{\left|\vec r-\vec v t\right|^3}+\beta r^2\int_0^{\theta'}\frac{\partial}{rc\,\partial t}E_i\left(\vec r-\vec v t,t\right)\sin\theta\,d\theta\right)\sin\theta'\,d\theta'\Bigg|_{t=0}$$

$$E_i\left(\vec r-\vec v t,t\right)=\frac{3}{2}\beta^2r^3\frac{q}{\left|\vec r-\vec v t\right|^5}\sin^2\theta'-\lambda r\frac{q}{\left|\vec r-\vec v t\right|^3} \tag{35a}$$

$$+\beta r^2\int_0^{\theta'}\left(\frac{3}{2}\beta^2r^3\frac{q}{\left|\vec r-\vec v t\right|^7}5\beta r\cos\theta'\sin^2\theta'-\frac{\lambda rq}{\left|\vec r-\vec v t\right|^5}3\beta r\cos\theta'\right)\sin\theta'\,d\theta'$$

$$+\beta^2r^4\int_0^{\theta}\frac{\partial}{rc\,\partial t}\left[\int_0^{\theta'}\frac{\partial}{rc\,\partial t}E_i\left(\vec r-\vec v t,t\right)\sin\theta\,d\theta\right]\sin\theta'\,d\theta'$$

In order to obtain the terms from the first iteration, one can finish taking the $t = 0$ and $\theta=\theta'$ to obtain

$$E_i\left(r\right)=\frac{q}{r^2}\frac{3\beta^2}{2}\sin^2\theta-\frac{\lambda q}{r^2}+\frac{q}{r^2}\frac{15\beta^4}{2}\int_0^{\theta}\sin^3\theta\,d\sin\theta-\frac{\lambda q3\beta^2}{r^2}\int_0^{\theta}\sin\theta\,d\sin\theta \tag{36}$$

$$=\frac{q}{r^2}\frac{3\beta^2}{2}\sin^2\theta-\frac{\lambda q}{r^2}+\frac{q}{r^2}\frac{15\beta^4}{8}\sin^4\theta-\frac{\lambda q}{r^2}\frac{3\beta^2}{2}\sin^2\theta$$

$$=\frac{q}{r^2}\left[\frac{3}{2}\beta^2\sin^2\theta+\frac{15\beta^4}{8}\sin^4\theta\right]-\frac{\lambda q}{r^2}\left[1+\frac{3}{2}\beta^2\sin^2\theta\right]$$

$$=E_o\left(r\right)\left[\frac{3}{2}\beta^2\sin^2\theta+\frac{15\beta^4}{8}\sin^4\theta\right]-\lambda E_o\left(r\right)\left[1+\frac{3}{2}\beta^2\sin^2\theta\right]$$

Solving for $\vec E\left(\vec r\right)=\vec E_o\left(\vec r\right)+\vec E_i\left(\vec r\right)$ obtain

$$\vec{E}\left(\vec{r}\right)=\vec{E}_o\left(\vec{r}\right)+\vec{E}_i\left(\vec{r}\right) \tag{37}$$

$$=\vec{E}_o\left(\vec{r}\right)\left[1+\frac{3}{2}\beta^2\sin^2\theta+\frac{15\beta^4}{8}\sin^4\theta+\ldots\right]-\lambda E_o\left(r\right)\left[1+\frac{3}{2}\beta^2\sin^2\theta+\ldots\right]$$

Using the binomial expansion

$$\left(1-\beta^2\sin^2\theta\right)^{-3/2}=1+\frac{3}{2}\beta^2\sin^2\theta+\frac{15}{8}\beta^4\sin^4\theta+\frac{35}{16}\beta^6\sin^6\theta+\cdots \tag{38}$$

to sum the series of iteration terms gives

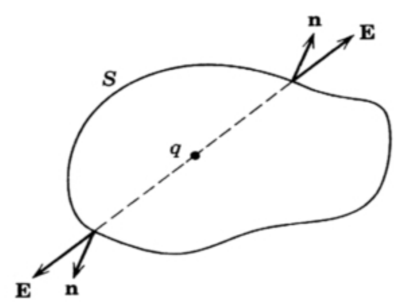
$$E\left(\vec{r}\right)=E_o\left(\vec{r}\right)+E_i\left(\vec{r}\right)=\vec{E}_o\left(\vec{r}\right)\frac{\left(1-\lambda\right)}{\left(1-\beta^2\sin^2\theta\right)^{3/2}} \tag{39}$$

Equation (39) gives the resultant self-consistent electric field of an elastic finite-size moving charged particle with total charge q as seen by an observer in his frame of reference.

In order to evaluate the constant λ one uses Gauss's law for electric charge [9a *p.* 4 or 9b *p.* 30 or 9c *p.* 27]. From Figure 4

Figure 4.
Geometry for Gauss's Law

$$\int_S \vec{E}\left(\vec{r}\right)\cdot\hat{n}\,da=4\pi q \tag{40}$$



Using a spherical surface centered about the charge distribution q with spherical coordinates and noting that $\vec{E}\left(\vec{r}\right)$ and $\hat{n}$ will then be in the same direction, equation (40) becomes

$$4\pi q=\int_0^{2\pi}\int_0^{\pi}\vec{E}_o\left(\vec{r}\right)\frac{\left(1-\lambda\right)}{\left(1-\beta^2\sin^2\theta\right)^{3/2}}r^2\sin\theta\,d\theta\,d\phi=4\pi q\,\frac{1-\lambda}{1-\beta^2} \tag{41}$$

where

$$\int_0^{\pi}\left(1-\beta^2\sin^2\theta\right)^{-3/2}\sin\theta\,d\theta=\frac{2}{1-\beta^2}\quad\text{and}\quad E_o=\frac{q}{r^2} \tag{42}$$

So λ must be equal to β^2 in order for the total flux of $\vec{E}\left(\vec{r}\right)$ to be conserved. Thus the expression for the self-consistent fields may be written as

$$\vec{E}\left(\vec{r}\right)=\vec{E}_o\left(\vec{r}\right)\frac{1-\beta^2}{\left(1-\beta^2\sin^2\theta\right)^{3/2}} \tag{43}$$

$$\vec{B}_i\left(\vec{r}\right)=\frac{\vec{v}}{c}\times\vec{E}\left(\vec{r}\right)$$

Equation (43) is the derived version of the $\vec{E}$ and $\vec{B}$ fields in the lab frame to order **v** in the Galilean transformation. This equation is precisely the same as one would obtain from Special Relativity Theory using the Lorentz transformation for the electric and magnetic fields observed in the observer's frame of reference for a point charge q passing by with relative uniform velocity **v** [9b *p.* 555 or 9c *p.* 560]. Although the mathematical expressions are the same, only this derivation is based upon causality and the fundamental empirical laws of electrodynamics using real particles with finite size and an internal charge structure and contact forces satisfying Mach's principle. *Perhaps the most significant observation of all is that the scheme of Special Relativity Theory only imitates the self-field effects to order* v *in the Galilean transformation. The terms involving acceleration and other higher order time derivatives are missing!*

Now the total electromagnetic force $\vec{F}$ exerted by the moving charge distribution on a test charge q' is

$$\begin{aligned}
\vec{F}\left(\vec{r},\vec{v},\ldots\right) &= q'\left(\vec{E}\left(\vec{r}\right)+\frac{\vec{v}}{c}\times\vec{B}_i\left(\vec{r}\right)\right) \qquad (44)\\
&= q'\left|\vec{E}_o\left(\vec{r}\right)\right|\frac{\left(1-\beta^2\right)}{\left(1-\beta^2\sin^2\theta\right)^{3/2}}\left[\left(1-\beta^2+\beta^2\cos^2\theta\right)\hat{r}-\left(\vec{\beta}\cdot\hat{r}\right)\hat{r}\times\left(\hat{r}\times\vec{\beta}\right)\right]\\
&= q'\left|\vec{E}_o\left(\vec{r}\right)\right|\frac{\left(1-\beta^2\right)}{\left(1-\beta^2\sin^2\theta\right)^{3/2}}\left[\left(1-\beta^2\sin^2\theta\right)\hat{r}-\left(\vec{\beta}\cdot\hat{r}\right)\hat{r}\times\left(\hat{r}\times\vec{\beta}\right)\right]\\
&= q'\left|\vec{E}_o\left(\vec{r}\right)\right|\frac{\left(1-\beta^2\right)}{\left(1-\beta^2\sin^2\theta\right)^{1/2}}-q'\left|\vec{E}_o\left(\vec{r}\right)\right|\frac{\left(1-\beta^2\right)\left(\vec{\beta}\cdot\hat{r}\right)\hat{r}\times\left(\hat{r}\times\vec{\beta}\right)}{\left(1-\beta^2\sin^2\theta\right)^{3/2}}
\end{aligned}$$

where the identities below were used.

$$\begin{aligned}
\frac{\vec{v}}{c}\times\left[\frac{\vec{v}}{c}\times\vec{E}_o\left(\vec{r}\right)\right] &= \frac{\vec{v}}{c}\cdot\vec{E}_o\left(\vec{r}\right)\frac{\vec{v}}{c}-\frac{\vec{v}^2}{c^2}\vec{E}_o\left(\vec{r}\right) \qquad (45)\\
&= \frac{\vec{v}}{c}\cdot\vec{E}_o\left(\vec{r}\right)\left[\frac{\left(\vec{v}\cdot\hat{r}\right)\hat{r}}{c}-\frac{\hat{r}\times\left(\hat{r}\times\vec{v}\right)}{c}\right]-\frac{\vec{v}^2}{c^2}\vec{E}_o\left(\vec{r}\right)
\end{aligned}$$

where

$$\begin{aligned}
\vec{v} &= \vec{v}-\left(\vec{v}\cdot\hat{r}\right)\hat{r}+\left(\vec{v}\cdot\hat{r}\right)\hat{r}=\left(\vec{v}\cdot\hat{r}\right)\hat{r}-\hat{r}\times\left(\hat{r}\times\vec{v}\right)\\
\hat{r}\times\left(\hat{r}\times\vec{v}\right) &= \left(\hat{r}\cdot\vec{v}\right)\hat{r}-\left(\hat{r}\cdot\hat{r}\right)\vec{v}=\left(\hat{r}\cdot\vec{v}\right)\hat{r}-\vec{v}
\end{aligned}$$

Equation (44) is a derived version of the extended Weber's force $\vec{F}_W$ for an elastic finite-size charged particle to order **v** in the Galilean transformation. The electromagnetic potential U corresponding to equation (44) for the Weber force that is accurate to order **v** in the Galilean transformation is

$$U=\frac{qq'}{R}\frac{\left(1-\beta^2\right)}{\left(1-\beta^2\sin^2\theta\right)^{1/2}}=qq'\frac{\left(1-\beta^2\right)}{\left[\vec{R}^2-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^2}{\vec{R}^2}\right]^{1/2}} \qquad (46)$$

Case for Relative Acceleration. Assuming that the electromagnetic potential is a regular function and well-behaved, one can obtain some of the acceleration terms of the force by treating the velocity as a function of time.

$$\vec{V}\cdot\vec{F}' = -\frac{dU\left(\vec{R},\vec{V},\ldots\right)}{dt} = -\frac{d}{dt}\left[\frac{qq'\left(1-\vec{\beta}^{2}\right)}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}}\right] = -qq'\left[\frac{\frac{d}{dt}\left(-\vec{\beta}^{2}\right)}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}}+\frac{\left(1-\vec{\beta}^{2}\right)\frac{-1}{2}\frac{d}{dt}\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}\right] \quad (47a)$$

$$= qq'\left[\frac{\frac{2}{c^{2}}\vec{V}\cdot\vec{A}}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}}+\frac{\left(1-\vec{\beta}^{2}\right)}{2}\frac{\left[2\vec{R}\cdot\vec{V}+\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}\left(\frac{-2}{\vec{R}^{4}}\right)\vec{R}\cdot\vec{V}\right]}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}+\frac{\left(1-\vec{\beta}^{2}\right)}{2}\frac{\frac{2\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)}{\vec{R}^{2}}\cdot\frac{d}{dt}\left(\vec{R}\left(\vec{R}\cdot\vec{\beta}\right)-\vec{\beta}\left(\vec{R}\cdot\vec{R}\right)\right)}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}\right] \quad (47b)$$

$$= qq'\left[\frac{\frac{2}{c^{2}}\vec{V}\cdot\vec{A}}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}}+\left(1-\vec{\beta}\right)\frac{\left[\vec{R}\cdot\vec{V}\left(1-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{4}}\right)\right]}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}+\left(1-\beta\right)\frac{\frac{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)}{\vec{R}^{2}}\left\{\vec{V}\left(\vec{R}\cdot\vec{\beta}\right)+\vec{R}\left(\vec{V}\cdot\vec{\beta}\right)+\vec{R}\left(\vec{R}\cdot\frac{\vec{A}}{c}\right)-\frac{\vec{A}}{c}\left(\vec{R}\cdot\vec{R}\right)-2\vec{\beta}\left(\vec{V}\cdot\vec{R}\right)\right\}}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}\right] \quad (47c)$$

$$= qq'\left[\frac{\left(1-\beta^{2}\right)\vec{V}\cdot\vec{R}+\frac{2\vec{R}^{2}}{c^{2}}\vec{V}\cdot\vec{A}}{\vec{R}^{2}\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}}+\left(1-\beta^{2}\right)\frac{\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}}{\vec{R}^{2}}\cdot\left\{-\frac{\vec{V}}{c}\left(\vec{V}\cdot\vec{R}\right)+\frac{\vec{R}}{c}\left(\vec{V}^{2}\right)+\vec{R}\times\left(\vec{R}\times\vec{A}\right)\right\}}{\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}\right] \quad (47d)$$

$$= qq'\left[\frac{\left(1-\beta^{2}\right)\vec{V}\cdot\vec{R}+\frac{2\vec{R}^{2}}{c^{2}}\vec{V}\cdot\vec{A}}{\vec{R}^{2}\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}}+\left(1-\beta^{2}\right)\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}\cdot\left\{-\frac{\vec{V}}{c}\left(\vec{V}\cdot\vec{R}\right)+\frac{\vec{R}}{c}\left(\vec{V}\cdot\vec{V}\right)+\left\{\vec{R}\left(\vec{R}\cdot\vec{\beta}\right)-\vec{\beta}\left(\vec{R}\cdot\vec{R}\right)\right\}\cdot\left\{+\vec{R}\times\left(\vec{R}\times\vec{A}\right)\right\}\right\}}{\vec{R}^{2}\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}\right] \quad (47e)$$

$$= qq'\left[\frac{\left(1-\beta^{2}\right)\vec{V}\cdot\vec{R}+\frac{2\vec{R}^{2}}{c^{2}}\vec{V}\cdot\vec{A}}{\vec{R}^{2}\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}}-\left(1-\beta^{2}\right)\frac{\left[\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}\cdot\vec{V}\left(\vec{\beta}\cdot\vec{R}\right)+\left\{\vec{R}\times\left(\vec{R}\times\vec{A}\right)\right\}\cdot\vec{V}\left(\frac{\vec{R}\cdot\vec{R}}{c}\right)\right]\cdot}{\vec{R}^{2}\left[\vec{R}^{2}-\frac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}\right] \quad (47f)$$

where the following vector identities were used

$$\vec{A}\times\left(\vec{B}\times\overline{C}\right)=\left(\vec{A}\cdot\vec{C}\right)\vec{B}-\left(\vec{A}\cdot\vec{B}\right)\vec{C}$$
$$\vec{R}\cdot\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)=\left(\vec{R}\cdot\vec{\beta}\right)\left(\vec{R}\cdot\vec{R}\right)-\left(\vec{R}\cdot\vec{R}\right)\left(\vec{R}\cdot\vec{\beta}\right)=0 \qquad (47g)$$
$$\vec{R}\cdot\vec{R}\times\left(\vec{R}\times\vec{A}\right)=\left(\vec{R}\cdot\vec{A}\right)\left(\vec{R}\cdot\vec{R}\right)-\left(\vec{R}\cdot\vec{R}\right)\left(\vec{R}\cdot\vec{A}\right)=0$$

Thus the force, with some of the acceleration terms in it, is

$$\vec{F}\left(\vec{R},\vec{V},\vec{A},\ldots\right)=q\,q'\frac{\left(1-\vec{\beta}^{2}\right)\vec{R}+\dfrac{2\vec{R}^{2}}{c^{2}}\vec{A}}{\vec{R}^{2}\left[\vec{R}^{2}-\dfrac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{1/2}} \qquad (48)$$
$$-q\,q'\left(\left(1-\vec{\beta}^{2}\right)\frac{\left(\vec{\beta}\cdot\vec{R}\right)\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)+\left(\vec{R}\cdot\vec{R}\right)\vec{R}\times\left(\vec{R}\times\dfrac{\vec{A}}{c^{2}}\right)}{\vec{R}^{2}\left[\vec{R}^{2}-\dfrac{\left\{\vec{R}\times\left(\vec{R}\times\vec{\beta}\right)\right\}^{2}}{\vec{R}^{2}}\right]^{3/2}}\right)$$

This force in the limit of constant velocity, *i.e.* $\vec{A}=0$, corresponds to the covariant relativistic electrodynamic force of equation (12) based on Maxwell's equations [9b *p.* 555 or 9c *p.* 560]. Note that the second group of terms in the equation for the force is responsible for the curling corkscrew type of motion of moving charges as seen in plasma currents in fluorescent lights and plasma lightning balls sold at many gift shops. Figure 5 shows the curling corkscrew motion of charge due to the electrodynamic force for constant velocity. In general there is a second smaller scale curling motion superimposed upon the first due to acceleration.

According to Ampere [15] the effect of a current flowing in a

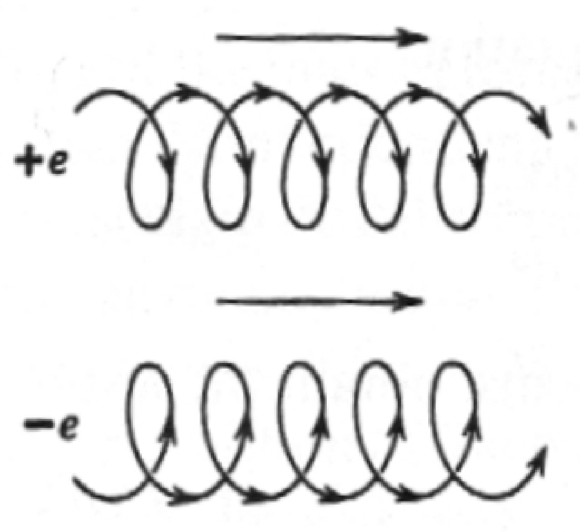


Figure 5. Corkscrew Motion of Charge Due to Electromagnetic Force

circuit twisted into small sinuosities is the same as if the circuit were smoothed out. For $\vec{v}^2/c^2 << 1$ and $aR/c^2 << 1$ this is true, but not for relativistic velocities $v/c \sim 1$. *Thus the force between two moving charges is increased due to the reduction of the effective distance R between the charges due to the spiraling motion of each charge according to the* $\vec{R}\times\left(\vec{R}\times\vec{v}/c\right)$ *term in the electromagnetic force.*

SUMMARY OF RESULTS

A universal electrodynamic force law for elastic finite-size charged particles was derived by simultaneously solving, by substitution, a complete set of the fundamental empirical laws of classical electrodynamics, *i.e.* Gauss's laws, Ampere's law, Faraday's law, and Lenz's law assuming Galilean invariance. The mathematics of the derivation and Hooper's experimental results require that the electrodynamic force be a local contact force based on the electromagnetic fields being an extension of the charges. The Lorentz force law is derived from Galilean invariance implying that it is not a fundamental law of electrodynamics.

This newly derived version of Weber's force law appears to be fully "relativistic" without any reference to Einstein's Special Relativity Theory or retarded fields. It accounts for relativistic effects including radiation (in Part 4).

This approach deals with a number of loose ends in electrodynamics. It derives the Lorentz force from Galilean invariance. It deals with real elastic particles with finite size and self fields. It does not use fictitious action-at-a-distance forces between point particles, but uses real contact forces based on the physical extension of charges via the charge's fields. It satisfies Mach's principle which Einstein was never able to do with special relativity by using Lenz's law. It uses the Galilean transformation based upon causality. It conserves energy and momentum 100% of the time compared to quantum electrodynamics which depends on the uncertainty principle to allow it to escape these conservation laws for brief periods of time. It only depends on the relative distance $\vec{R}$, the relative velocity $\vec{V}$, the relative acceleration $\vec{A}$ and the relative $d\vec{A}'/dt$ between the interacting charges implying that the force has the same value for all observers irrespective of their states of motion or of their inertial frame of reference. It is a proper relativistic theory according to the classical definition, because it only depends on relative coordinates.

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