Derivation of the Universal Force Law—Part 2

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Abstract. A new universal electromagnetic force law for real finite-size elastic charged particles is derived by solving simultaneously the fundamental empirical laws of classical electrodynamics, *i.e.* Gauss's laws, Ampère's generalized law, Faraday's law, and Lenz's law assuming Galilean invariance. This derived version of the electromagnetic force law incorporates the effects of the self-fields of real finite-size elastic particles as observed in particle scattering experiments. It can account for gravity, inertia, and relativistic effects including radiation. The non-radial terms of the force law explain the experimentally observed curling of plasma currents, the tilting of the orbits of the planets with respect to the equatorial plane of the sun, and certain inertial gyroscope motions. The derived force law satisfies Newton's third law, conservation of energy and momentum, conservation of charge, and Mach's Principle. The mathematical properties of equations for the fundamental empirical laws and also Hooper's experimental data showing that the fields of a moving charge move with the charge require that the electrodynamic force be a contact force based on field extensions of the charge instead of action-at-a-distance. The Lorentz force is derived as a consequence of Galilean invariance. The most general form of the force law, derived using all the higher order terms of the Galilean transformation, is assumed to be exact for all phenomena on all size scales. Arguments are given that this force law is superior to all previous force laws, *i.e.* relativistic quantum electrodynamic, gravitational, inertial, strong interaction and weak interaction force laws.

Derivation of Electrodynamic Force Law for Finite-Size Particles. In the derivation that follows [10, 11a, 11b, 11c] it is assumed that the universal electrodynamic force law is based upon six empirical laws of electrodynamics, *i.e.*

Ampère's Law $$\vec{B}_i\left(\vec{r},t\right)=\frac{\vec{v}}{c}\times\vec{E}_o\left(\vec{r}\,',t'\right) \quad \text{(Grassman form)}$$

Faraday's Law $$\int \vec{E}\left(\vec{r}\,',t'\right)dl' = -\frac{1}{c}\frac{d}{dt}\left[\int \vec{B}\left(\vec{r},t\right)\cdot\hat{n}\,da\right]$$

Gauss's Laws $$\int \vec{E}\left(\vec{r},t\right)\cdot\hat{n}\,da = 4\pi q \quad \text{and} \quad \nabla\cdot\vec{B}\left(\vec{r},t\right)=0$$

Lenz's Law $$\vec{E}\left(\vec{r},t\right)\propto -\vec{E}_o\left(\vec{r},t\right) \quad \text{where} \quad \vec{E}\left(\vec{r},t\right)=\vec{E}_o\left(\vec{r},t\right)+\vec{E}_i\left(\vec{r},t\right)$$

Lorentz's Law $$\vec{F}=q\,\vec{E}\left(\vec{r},t\right)-\frac{\vec{v}}{c}\times\vec{B}_i\left(\vec{r},t\right)$$

Note that relativistic type notation is used and that both Ampère's law and Faraday's law involve the observer's reference frame and a moving frame of reference that are described by the Galilean transformation (see Figure 1):

$$\vec{r}\,' = r' - \frac{at^2}{2} - \ldots \quad \text{and} \quad t' = t$$

In the past physicists have willfully discarded the Galilean transformation in favor of the relativistic Lorentz transformation to relate electromagnetic fields in the two frames. This derivation will show that this illogical procedure was totally unnecessary and resulted in the creation of the superfluous theory of special relativity.

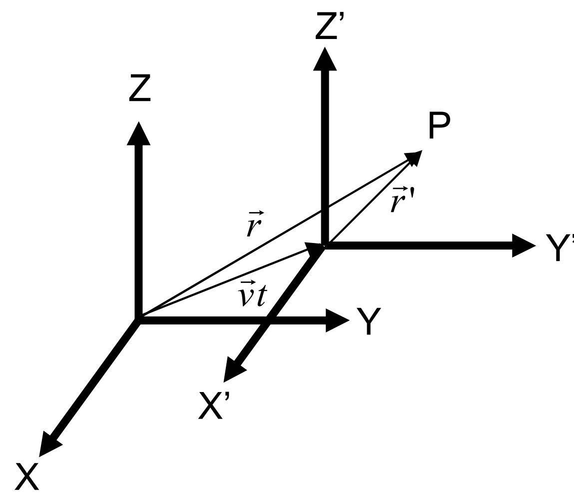

Figure 1. Coordinates of Particle at Point P

In this paper the empirical equations of electrodynamics are solved simultaneously by the method of substitution using the Galilean transformation to eliminate references to the primed reference frame. The resulting electric and magnetic fields in the observer's frame of reference will be derived for an elementary particle with an arbitrary finite-size elastic charge distribution of total charge q moving with relative velocity $\vec{V}$, acceleration $\vec{A} = d\vec{V}/dt$ *etc.*

When this paper was first written, it was not known that the force law derived was the universal force law. This fact was only appreciated after the force of gravity and the force of inertia were derived from it. At that point the derivation became a proof against all previous force laws as well as special and general relativity theory. Also at that point many theoreticians objected to the use of Grassman's form or version of Ampère's law. Note that Maxwell also used the Grassman version of Ampère's law.

In the appendix of the original version of this paper is a derivation of Ampère's force law between elements of current loops and the generalized version of Ampère's force law and the proof that both satisfy Newton's third law. Also included is a derivation of the Biot-Savart law and the Grassman force law from the generalized Ampère force law. This derivation shows that both the Biot-Savart law and the Grassman force law satisfy Newton's third law when one notes that the additional term in the generalized Ampère force law that is missing in the Grassman force law gives no contribution for closed current loops. *But of course, Ampère's force law only applies to closed current loops!* Since this is now such an important issue, the appendix of the original paper will be reproduced here in the text.

Appendix: Derivation of the Biot-Savart Law and the Grassman Law from Ampère's Law

The magnetic effect of an electric current was discovered in 1819 by the Danish physicist and chemist, Hans Christian Oersted (1777-1851)[16]. He found that a compass needle placed under an electric current takes up a direction perpendicular to that of the current.

This observation, which was the first to link electricity and magnetism, stimulated the French physicist and mathematician Andre Marie Ampère (1775-1836) to perform many experiments exploring the implications of this observation. Ampère observed the action of an electric current upon a magnet, the action of a magnet upon an electric current, and the action of one electric current upon another electric current. These actions and reactions were found to be consistent with Newton's third law ("to every action there is always opposed an equal reaction").

Ampère [17] performed a series of four experiments and found that the force between two current elements in current loops obey the following laws:

1. The effect of a current is reversed when the direction of the current is reversed.

2. The effect of a current flowing in a circuit twisted into small sinuosities is the same as if the circuit were smoothed out.

3. The force exerted by a closed circuit on an element of another circuit is at right angles to the latter.

4. The force between two elements of circuits is unaffected when all linear dimensions are increased proportionately and the current strengths remain unaltered.

From these four laws and the assumption that the force between two elements of circuits act along the line joining them, Ampère obtained a mathematical expression for the force. The deduction of the force law was made as follows (see Figure 2):

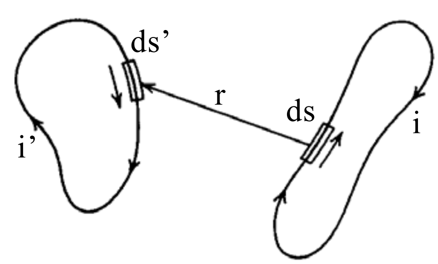

Figure 2.

Ampère's Force Between Current Elements

From law (2) the effect of **ds** on **ds'** is the vector sum of the effects on **ds'**. From law (1) which supports Newton's third law of action and reaction plus the assumption that the force is linear and homogeneous in **ds** and **ds'**, the simplest general formula must be

$$\vec{F}_{ij} = ii'\left[r\left(d\vec{s}_i \cdot d\vec{s}'_j\right)\Phi_1(r) + d\vec{s}_i\left(d\vec{s}'_j \cdot \vec{r}\right)\Phi_2(r)\right.$$
$$\left. + d\vec{s}'_j\left(d\vec{s}_i \cdot \vec{r}\right)\Phi_3(r) + r\left(d\vec{s}_i \cdot \vec{r}\right)\left(d\vec{s}'_j \cdot \vec{r}\right)\Psi(r)\right] \quad \text{(A1)}$$

where $\Phi_1(r)$, $\Phi_2(r)$, $\Phi_3(r)$, $\Psi(r)$ are functions of r still to be determined.

The assumption that the force between two elements of the circuits acts along the line joining them implies that the $\Phi_2(r)$ and $\Phi_3(r)$ terms should be dropped since they are not proportional to the vector **r** giving

$$\vec{F}_{ij} = ii'\vec{r}\left[\left(\Phi_1(r)d\vec{s}_i \cdot d\vec{s}'_j\right) + \Psi(r)\left(d\vec{s}_i \cdot \vec{r}\right)\left(d\vec{s}'_j \cdot \vec{r}\right)\right] \quad \text{(A2)}$$

From law (4) the force $\vec{F}_{ij}$ is unaffected when $d\vec{s}_i, d\vec{s}_j$, and $\vec{r}$ are all changed by the same factor. The simplest forms for $\Phi_1(r)$ and $\Psi(r)$ for this to be true are

$$\Phi_1(r) = \frac{\mathrm{A}}{r^3} \quad \text{and} \quad \Psi(r) = \frac{\mathrm{B}}{r^5} \quad \text{(A3)}$$

giving

$$\vec{F}_{ij} = ii'\vec{r}\left[\frac{\mathrm{A}}{r^3}\right]\left(d\vec{s}_i \cdot d\vec{s}'_j\right) + \frac{\mathrm{B}}{r^5}\left(d\vec{s}_i \cdot \vec{r}\right)\left(d\vec{s}'_j \cdot \vec{r}\right) \quad \text{(A4)}$$

where A and B denote constants still to be determined.

From law (3) the projection of $\vec{F}_{ij}$ along $d\vec{s}'$ must vanish when integrated around the circuit s, *i.e.*

$$\oint\left(\vec{F}_{ij} \cdot d\vec{s}'_j\right)d\vec{s}'_i = 0$$

$$= ii'\oint\left[\frac{A}{r^3}\left(d\vec{s}_i \cdot d\vec{s}'_j\right)\left(\vec{r} \cdot d\vec{s}'_j\right) + \frac{\mathrm{B}}{r^5}\left(d\vec{s}_i \cdot \vec{r}\right)\left(d\vec{s}'_j \cdot \vec{r}\right)^2\right]d\vec{s} \quad \text{(A5)}$$

For the limiting case $d\vec{s}_i = -d\vec{r}$ where $d\vec{s}_i$ is fixed, one can obtain a relationship between A and B, *i.e.*

$$0 = -ii'\oint\left[\frac{\mathrm{A}}{2r^3}d\left(d\vec{s}'_j \cdot \vec{r}\right)^2 + \frac{\mathrm{B}}{r^4}\left(d\vec{s}'_j \cdot \vec{r}\right)^2 d\vec{r}\right]d\vec{s} \quad \text{(A7)}$$

In order for this to be true the integrand must be a complete differential such that

$$d\left(\frac{\mathrm{A}}{2r^3}\right) = -\frac{3}{2}\frac{\mathrm{A}}{r^4}dr = \frac{\mathrm{B}}{r^4}dr \quad \text{(A8)}$$

giving

$$\mathrm{B} = -\frac{3}{2}\mathrm{A} \quad \text{(A9)}$$

Thus the Ampère law for the force between two currents loops is

$$\vec{F}_{ij} = -\frac{A}{2} ii'\vec{r}\left[\frac{2}{r^3}\left(d\vec{s}_i \cdot d\vec{s}'_j\right) - \frac{3}{r^5}\left(d\vec{s}_i \cdot \vec{r}\right)\left(d\vec{s}'_j \cdot \vec{r}\right)\right] \qquad (A10)$$

As Whittaker[18] points out there is a weakness in Ampère's law. The weakness is the assumption that the force between two circuit elements acts only along the line joining them. In the analogous case of the action between magnetic molecules, the experimentally observed force is not entirely directed along the line joining the molecules. Also Helmholtz [19] assumes that the interaction between two current elements is derivable from a potential, like Weber's potential, and this entails the existence of a couple in addition to a force along the line joining the elements. Thus in order to obtain the more general force law between current elements, one must include all of the terms linear and homogeneous in $d\vec{s}$ and $d\vec{s}'$ in equation (A1).

Now Ampère has already obtained one set of terms that satisfies Newton's third law. In order to get an additional set of terms that satisfy Newton's third law for the forces between current elements that are not directed along the line between them, a second full set of terms need to be added. Noting that these terms must also form a complete differential, the more general equation for $\vec{F}_{ij}$ is

$$\begin{aligned}\vec{F}_{ij} = &-\frac{ii'\vec{r}}{c}\left[\frac{2}{r^3}\left(d\vec{s}_i \cdot d\vec{s}'_j\right) - \frac{3}{r^5}\left(d\vec{s}_i \cdot \vec{r}\right)\left(d\vec{s}'_j \cdot \vec{r}\right)\right] \\ &+ \chi(r)\left(d\vec{s}'_j \cdot \vec{r}\right)d\vec{s}_i + \chi(r)\left(d\vec{s}_i \cdot \vec{r}\right)d\vec{s}'_j \\ &+ \chi(r)\left(d\vec{s}_i \cdot d\vec{s}'_j\right)\vec{r} + \frac{1}{r}\chi'(r)\left(d\vec{s}_i \cdot \vec{r}\right)\left(d\vec{s}'_j \cdot \vec{r}\right)\vec{r}\end{aligned} \qquad (A11)$$

where for CGS units the constant A/2 = 1/c.

The simplest solution for $\chi(r)$ that satisfies Ampère's four laws for current elements is

$$\chi(r) = \frac{ii'}{cr^3} \qquad (A12)$$

Substituting in equation (A11) one notes some cancellation of terms to obtain

$$F_{ij} = \frac{ii'}{cr^3}\left[\left(d\vec{s}_i \cdot \vec{r}\right)d\vec{s}'_j + \left(d\vec{s}'_j \cdot \vec{r}\right)d\vec{s}_i - \left(d\vec{s}_i \cdot d\vec{s}'_j\right)\vec{r}\right] \qquad (A13)$$

For forces between current loops, if one calculates the force $\vec{F}_i$ on the current element $d\vec{s}_i$ due to the entire *j'* current loop,

$$\vec{F}_i = \frac{ii'}{cr^3}\left[\left(d\vec{s}_i \cdot \vec{r}\right)\oint d\vec{s}'_j + \oint\left(d\vec{s}'_j \cdot \vec{r}\right)d\vec{s}_i - \oint\left(d\vec{s}_i \cdot d\vec{s}'_j\right)\vec{r}\right] \qquad (A14)$$

Then the first term $\left(d\vec{s} \cdot \vec{r}\right)\oint d\vec{s}'_j$ evaluates to zero, because the *j'* loop is a closed loop. Thus this term does not make any effective contribution to the force. In other words Ampère's general force law for forces between current loops is not mathematically unique. Note that Grassman [20] omits this term by making the assumption that two current elements in the same straight line have no mutual action or longitudinal force between them. However it is necessary to see this term in order to understand that Newton's third law is satisfied by both Grassman and Ampère force laws. Assis [23,24] has shown that Ampère's and Grassman's expressions for the force always yield the same results when considering the net force on any current element of a closed circuit of arbitrary shape.

The general form of the force law of equation (A13) can be written in terms of a triple vector product

$$\vec{F}_{ij} = i\,d\vec{s}_i \times \frac{i'\,d\vec{s}'_j \times \vec{r}_{ij}}{c r_{ij}^3} = i\,d\vec{s}_i \times d\vec{B}_{ij'} \tag{A15}$$

where

$$d\vec{B}_{ij'} = i'\frac{d\vec{s}'_j \times \vec{r}_{ij}}{c r_{ij}^3} \tag{A16}$$

Heaviside [21,22] noted in 1888 that the original Ampère law in the form of equation (A10) was not actually being used in practice. He suggested that the Father of Electrodynamics have his name attached to the more general force law of equation (A15) that is based on his experimental observations. Unfortunately Grassman and to some extent Biot and Savart also had their name associated with various forms of this equation. There has been confusion ever since. Maxwell attributed his differential equation which results from the more general form of the force law to Ampère.

Equations (A15) and (A16) only have meaning as one element of a sum over a continuous set where the sum represents the magnetic induction of a current loop or circuit. The continuity equation

$$\nabla \cdot J = 0 \tag{A17}$$

is not satisfied for the current element $i\,d\vec{s}$ by itself, because the current comes from nowhere and disappears after traversing the length $d\vec{s}$.

The German mathematician Herman Gunther Grassman (1809-1877) rewrote equations (A15) and (A16) by replacing $i\,d\vec{s}$ by $-q\vec{v}$ where q is the charge in the segment $d\vec{s}$ and $\vec{v}$ is its velocity. Integrating both sides of the equation obtain

$$\vec{F}_{ij} = q_i\left(\vec{v}_i \times \vec{B}_{ij}\right) \tag{A18}$$

and

$$\vec{B}_{ij} = q_j\frac{\left(\vec{v}_j \times \vec{r}_{ij}\right)}{c\left|\vec{r}_{ij}\right|^3} \tag{A19}$$

Equation (A18) is known as the Grassman force law. Grassman's substitution of $-q\vec{v}$ for $i\,d\vec{s}$ is based upon the following reasoning. If we let Q be the total charge in the circuit, and assume that the total charge of a closed system or circuit remains constant then

$$\frac{dQ}{dt} = 0 \quad \text{with} \quad Q = \sum_{i=1}^{n} q_i \, ds_i \tag{A20}$$

Taking the derivative explicitly with Q one obtains

$$\frac{dQ}{dt} = \sum_{i=1}^{n} \left(ds_i \frac{dq_i}{dt} + q_i \frac{ds_i}{dt} \right) = 0 \tag{A21}$$

Requiring each term in the series i = 1 to n to individually be equal to 0 gives the relation

$$ds_i \frac{dq_i}{dt} = -q_i \frac{ds_i}{dt} \tag{A22}$$

or in general

$$I_i \, d\vec{s}_i = -\vec{v}_i \, q_i \tag{A23}$$

Thus it appears that on the most elementary basis Grassman's force law of equations (A18) and (A19) applies to the forces between elements of charges in complete loops or circuits and not arbitrary point charges. If one does not include all the charge elements of the complete circuit, one is not using the Grassman law correctly. This implies that Maxwell's equations and the covariant formulation of electrodynamics do not apply to point particles, as generally assumed, but only to charge elements of complete circuits or loops.

Using the Grassman form of the generalized Ampère force law of equation (A19) for a single point element of charge q moving with a relative velocity **v**, the induced flux density $\vec{B}_i(\vec{r},t)$ will be

$$\vec{B}_i(\vec{r},t) = q \frac{\vec{v} \times \vec{r}\,'}{c\left|\vec{r}\,'\right|^3} = \frac{\vec{v}}{c} \times \vec{E}_0 \left(\vec{r}\,',t'\right) \tag{15}$$

where the more familiar relativistic type notation of Figure 1 has been used for clarity. Note that equation (15) for a point element of charge gives the transformation of the $\vec{E}_0(\vec{r}\,',t')$ field in the moving frame of reference to the induced field $\vec{B}_i(\vec{r},t)$ in the observer's frame of reference. This is assumed valid for all velocities v' whether constant in time or changing, and will be used to obtain the fields of a charged particle with internal charge distribution.

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